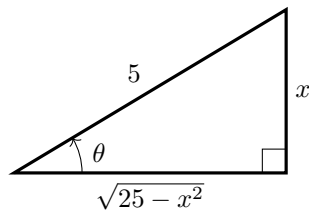


Question 2

Evaluate using a trigonometric substitution:

$$\int \frac{x^2}{\sqrt{25 - x^2}} dx.$$

Question 2 – Solution



The radical has the form $\sqrt{a^2 - x^2}$, so let

$$x = 5 \sin \theta, \quad dx = 5 \cos \theta \, d\theta.$$

Choose $-\pi/2 < \theta < \pi/2$, so $\cos \theta > 0$. Then

$$\sqrt{25 - x^2} = \sqrt{25 - 25 \sin^2 \theta} = 5 \cos \theta.$$

Substitution gives

$$\begin{aligned} I &= \int \frac{25 \sin^2 \theta}{5 \cos \theta} (5 \cos \theta \, d\theta) \\ &= 25 \int \sin^2 \theta \, d\theta \\ &= \frac{25}{2} \int (1 - \cos 2\theta) \, d\theta \\ &= \frac{25}{2} \theta - \frac{25}{4} \sin 2\theta + C. \end{aligned}$$

Since $\theta = \arcsin(x/5)$ and

$$\sin 2\theta = 2 \sin \theta \cos \theta = 2 \left(\frac{x}{5} \right) \left(\frac{\sqrt{25 - x^2}}{5} \right),$$

we obtain

$$\boxed{I = \frac{25}{2} \arcsin \left(\frac{x}{5} \right) - \frac{x}{2} \sqrt{25 - x^2} + C}.$$