

Question 10

A signal is defined by

$$s(t) = \sin(3t) + \sin(5t).$$

Compute its energy over $0 \leq t \leq 2\pi$:

$$E = \int_0^{2\pi} s^2(t) dt.$$

Question 10 – Solution

First expand the square:

$$\begin{aligned} E &= \int_0^{2\pi} (\sin(3t) + \sin(5t))^2 dt \\ &= \int_0^{2\pi} [\sin^2(3t) + 2\sin(3t)\sin(5t) + \sin^2(5t)] dt. \end{aligned} \tag{1}$$

For the cross term, use

$$2\sin(A)\sin(B) = \cos(A - B) - \cos(A + B).$$

Thus,

$$\begin{aligned} 2 \int_0^{2\pi} \sin(3t)\sin(5t) dt &= \int_0^{2\pi} (\cos(2t) - \cos(8t)) dt \\ &= \left[\frac{1}{2} \sin(2t) - \frac{1}{8} \sin(8t) \right]_0^{2\pi} = 0. \end{aligned}$$

For each squared term, use $\sin^2(kt) = \frac{1}{2}(1 - \cos(2kt))$:

$$\begin{aligned} \int_0^{2\pi} \sin^2(kt) dt &= \frac{1}{2} \int_0^{2\pi} (1 - \cos(2kt)) dt \\ &= \frac{1}{2} \left[t - \frac{\sin(2kt)}{2k} \right]_0^{2\pi} = \pi \end{aligned}$$

for any positive integer k . In particular, the $k = 3$ and $k = 5$ terms each contribute π . Substituting into equation (1),

$$E = \pi + 0 + \pi = 2\pi.$$

Therefore,

$$\boxed{E = 2\pi}.$$