

Question 9

Find constants A and B such that

$$\cos^4(x) = A + B \cos(2x) + \frac{1}{8} \cos(4x).$$

Then use the identity to evaluate $\int \cos^4(x) \, dx$.

Question 9 – Solution

Begin with the power-reduction identity

$$\cos^2(x) = \frac{1 + \cos(2x)}{2}.$$

Squaring both sides,

$$\begin{aligned}\cos^4(x) &= \left(\frac{1 + \cos(2x)}{2} \right)^2 \\ &= \frac{1}{4} (1 + 2 \cos(2x) + \cos^2(2x)) .\end{aligned}$$

Apply power reduction again:

$$\cos^2(2x) = \frac{1 + \cos(4x)}{2}.$$

Therefore,

$$\begin{aligned}\cos^4(x) &= \frac{1}{4} \left(1 + 2 \cos(2x) + \frac{1 + \cos(4x)}{2} \right) \\ &= \frac{3}{8} + \frac{1}{2} \cos(2x) + \frac{1}{8} \cos(4x).\end{aligned}$$

Matching this with the required form gives

$$\boxed{A = \frac{3}{8}, \quad B = \frac{1}{2}}.$$

Now integrate the identity term by term:

$$\begin{aligned}\int \cos^4(x) dx &= \int \left(\frac{3}{8} + \frac{1}{2} \cos(2x) + \frac{1}{8} \cos(4x) \right) dx \\ &= \frac{3x}{8} + \frac{1}{4} \sin(2x) + \frac{1}{32} \sin(4x) + C.\end{aligned}$$

Hence,

$$\boxed{\int \cos^4(x) dx = \frac{3x}{8} + \frac{1}{4} \sin(2x) + \frac{1}{32} \sin(4x) + C}.$$