

Question 6

Use a product-to-sum identity to evaluate

$$\int \sin(5x) \cos(2x) \, dx.$$

Question 6 – Solution

Use the product-to-sum identity

$$\sin(A) \cos(B) = \frac{1}{2} [\sin(A + B) + \sin(A - B)].$$

With $A = 5x$ and $B = 2x$,

$$\sin(5x) \cos(2x) = \frac{1}{2} [\sin(7x) + \sin(3x)].$$

Therefore,

$$\begin{aligned} \int \sin(5x) \cos(2x) dx &= \frac{1}{2} \int [\sin(7x) + \sin(3x)] dx \\ &= \frac{1}{2} \left(-\frac{1}{7} \cos(7x) - \frac{1}{3} \cos(3x) \right) + C \\ &= -\frac{1}{14} \cos(7x) - \frac{1}{6} \cos(3x) + C. \end{aligned}$$

Hence,

$$\boxed{\int \sin(5x) \cos(2x) dx = -\frac{1}{14} \cos(7x) - \frac{1}{6} \cos(3x) + C.}$$

Differentiating gives

$$\frac{1}{2} \sin(7x) + \frac{1}{2} \sin(3x) = \sin(5x) \cos(2x),$$

confirming the answer by the same product-to-sum identity.