

Question 5

Evaluate

$$\int \sec^5(x) \tan^3(x) \, dx$$

by saving a factor of $\sec(x) \tan(x)$ for substitution.

Question 5 – Solution

Separate one factor of $\sec(x) \tan(x)$:

$$\sec^5(x) \tan^3(x) = \sec^4(x) \tan^2(x) (\sec(x) \tan(x)).$$

Use $\tan^2(x) = \sec^2(x) - 1$ to rewrite the remaining tangent power:

$$\sec^4(x) \tan^2(x) = \sec^4(x) (\sec^2(x) - 1).$$

Now let

$$u = \sec(x), \quad du = \sec(x) \tan(x) dx.$$

The integral becomes

$$\begin{aligned} \int u^4(u^2 - 1) du &= \int (u^6 - u^4) du \\ &= \frac{u^7}{7} - \frac{u^5}{5} + C. \end{aligned}$$

Substituting $u = \sec(x)$ gives

$$\boxed{\int \sec^5(x) \tan^3(x) dx = \frac{1}{7} \sec^7(x) - \frac{1}{5} \sec^5(x) + C}.$$

Saving $\sec(x) \tan(x)$ is useful because it supplies du , while the identity converts all remaining factors into powers of u .