

Question 4

Evaluate

$$\int \tan^7(x) \sec^2(x) \, dx$$

without converting the integrand entirely to sines and cosines.

Question 4 – Solution

The factor $\sec^2(x) dx$ is exactly the differential of $\tan(x)$, so let

$$u = \tan(x), \quad du = \sec^2(x) dx.$$

Then

$$\begin{aligned} \int \tan^7(x) \sec^2(x) dx &= \int u^7 du \\ &= \frac{u^8}{8} + C. \end{aligned}$$

Substitute $u = \tan(x)$:

$$\boxed{\int \tan^7(x) \sec^2(x) dx = \frac{1}{8} \tan^8(x) + C}.$$

As a check,

$$\begin{aligned} \frac{d}{dx} \left[\frac{1}{8} \tan^8(x) \right] &= \frac{1}{8} \cdot 8 \tan^7(x) \sec^2(x) \\ &= \tan^7(x) \sec^2(x), \end{aligned}$$

which is the original integrand.