

Question 3

Find the average value of

$$f(x) = \sin^4(x)$$

over one full period.

Question 3 – Solution

We may use the interval $[0, 2\pi]$, whose length is 2π . The average-value formula is

$$f_{\text{avg}} = \frac{1}{2\pi} \int_0^{2\pi} \sin^4(x) dx.$$

Apply the power-reduction identity twice:

$$\begin{aligned} \sin^4(x) &= \left(\frac{1 - \cos(2x)}{2} \right)^2 \\ &= \frac{1}{4} (1 - 2\cos(2x) + \cos^2(2x)) \\ &= \frac{1}{4} \left(1 - 2\cos(2x) + \frac{1 + \cos(4x)}{2} \right) \\ &= \frac{3 - 4\cos(2x) + \cos(4x)}{8}. \end{aligned}$$

Thus,

$$\begin{aligned} f_{\text{avg}} &= \frac{1}{2\pi} \int_0^{2\pi} \frac{3 - 4\cos(2x) + \cos(4x)}{8} dx \\ &= \frac{1}{16\pi} \left[3x - 2\sin(2x) + \frac{1}{4}\sin(4x) \right]_0^{2\pi} \\ &= \frac{1}{16\pi} (6\pi) = \frac{3}{8}. \end{aligned}$$

The sine terms vanish because their values are zero at both endpoints. Therefore,

$$\boxed{f_{\text{avg}} = \frac{3}{8}}.$$