

Question 1

Evaluate

$$\int \sin^5(x) \cos^4(x) dx.$$

Explain why separating one factor of $\sin(x)$ is an efficient strategy.

Question 1 – Solution

The power of sine is odd, so separate one sine factor:

$$\sin^5(x) = \sin^4(x) \sin(x).$$

The remaining even power can be converted using $\sin^2(x) = 1 - \cos^2(x)$:

$$\sin^4(x) = (1 - \cos^2(x))^2.$$

Thus,

$$\int \sin^5(x) \cos^4(x) dx = \int (1 - \cos^2(x))^2 \cos^4(x) \sin(x) dx.$$

Let

$$u = \cos(x), \quad du = -\sin(x) dx.$$

Then the integral becomes

$$\begin{aligned} - \int (1 - u^2)^2 u^4 du &= - \int (1 - 2u^2 + u^4) u^4 du \\ &= - \int (u^4 - 2u^6 + u^8) du \\ &= -\frac{u^5}{5} + \frac{2u^7}{7} - \frac{u^9}{9} + C. \end{aligned}$$

Substituting $u = \cos(x)$ gives

$$\boxed{\int \sin^5(x) \cos^4(x) dx = -\frac{\cos^5(x)}{5} + \frac{2 \cos^7(x)}{7} - \frac{\cos^9(x)}{9} + C.}$$

Separating one sine factor is efficient because it supplies the differential $-\sin(x) dx$ while every remaining factor becomes a polynomial in u .