

Question 10

For Simpson's Rule on $[0, \pi]$, suppose

$$|f^{(4)}(x)| \leq 1.$$

Find the smallest even n that guarantees error at most 10^{-6} .

Question 10 – Solution

Step 1: Write Simpson's error bound.

$$|E_S| \leq \frac{K(b-a)^5}{180n^4}.$$

Here $K = 1$ and $b - a = \pi$, so

$$|E_S| \leq \frac{\pi^5}{180n^4}.$$

Step 2: Require the bound to be at most 10^{-6} .

$$\begin{aligned}\frac{\pi^5}{180n^4} &\leq 10^{-6}, \\ n^4 &\geq \frac{\pi^5}{180 \cdot 10^{-6}}, \\ n &\geq \left(\frac{\pi^5}{180 \cdot 10^{-6}} \right)^{1/4} \\ &\approx 36.109.\end{aligned}$$

Step 3: Enforce Simpson's even- n requirement. The first even integer at least 36.109 is 38.

$$\boxed{n = 38}$$