

Question 7

Show that Simpson's Rule with $n = 2$ is exact for

$$\int_0^2 x^3 dx.$$

Question 7 – Solution

Step 1: Find the width and nodes.

$$h = \frac{2-0}{2} = 1, \quad x_0 = 0, \ x_1 = 1, \ x_2 = 2.$$

Step 2: Apply Simpson's Rule.

$$\begin{aligned} S_2 &= \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)] \\ &= \frac{1}{3}[0^3 + 4(1^3) + 2^3] \\ &= \frac{1}{3}(0 + 4 + 8) = 4. \end{aligned}$$

Step 3: Evaluate the exact integral.

$$\begin{aligned} I &= \int_0^2 x^3 dx = \left[\frac{x^4}{4} \right]_0^2 \\ &= \frac{16}{4} - 0 = 4. \end{aligned}$$

Since $S_2 = I$, Simpson's Rule is exact here.

$$\boxed{S_2 = I = 4}$$