

Question 6

How large must n be for T_n to approximate

$$\int_0^1 e^x dx$$

within 10^{-4} ? Use $|f''(x)| \leq e$.

Question 6 – Solution

Step 1: Write the trapezoidal error bound.

$$|E_T| \leq \frac{K(b-a)^3}{12n^2}.$$

Here $K = e$ and $b - a = 1$, so

$$|E_T| \leq \frac{e}{12n^2}.$$

Step 2: Require the bound to be at most 10^{-4} .

$$\frac{e}{12n^2} \leq 10^{-4},$$

$$e \leq 0.0012n^2,$$

$$n^2 \geq \frac{e}{0.0012} \approx 2265.23,$$

$$n \geq \sqrt{2265.23} \approx 47.594.$$

Step 3: Choose the smallest integer.

$$\boxed{n = 48}$$