

**Question 5**

For  $\int_0^2 x^4 dx$ , determine whether  $T_n$  and  $M_n$  are overestimates or underestimates. Use concavity.

**Question 5 – Solution**

**Step 1: Determine concavity.** For  $f(x) = x^4$ ,

$$f'(x) = 4x^3, \quad f''(x) = 12x^2.$$

On  $[0, 2]$ ,  $f''(x) \geq 0$ , so the graph is concave up.

**Step 2: Interpret the trapezoidal approximation.** For a concave-up function, each secant line lies above the graph. Therefore,

$$T_n > \int_0^2 x^4 dx.$$

**Step 3: Interpret the midpoint approximation.** For a concave-up function, midpoint rectangles lie below the graph on average. Therefore,

$$M_n < \int_0^2 x^4 dx.$$

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| $T_n > I, \quad M_n < I$ |
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