

Question 4

For increasing f , explain why

$$L_n \leq \int_a^b f(x) \, dx \leq R_n.$$

Then bound $\int_0^1 e^x \, dx$ using $n = 2$.

Question 4 – Solution

Step 1: Justify the inequality. On each subinterval, an increasing function satisfies

$$f(x_{i-1}) \leq f(x) \leq f(x_i).$$

Multiplying by $\Delta x > 0$ and adding over all subintervals gives

$$L_n \leq \int_a^b f(x) dx \leq R_n.$$

Step 2: Find the width and nodes.

$$\Delta x = \frac{1-0}{2} = \frac{1}{2}, \quad x_0 = 0, \quad x_1 = \frac{1}{2}, \quad x_2 = 1.$$

Step 3: Compute both sums.

$$L_2 = \frac{1}{2}[f(0) + f(1/2)] = \frac{1}{2}(1 + e^{1/2}) \approx 1.3244,$$

$$R_2 = \frac{1}{2}[f(1/2) + f(1)] = \frac{1}{2}(e^{1/2} + e) \approx 2.1835.$$

Step 4: State the bound.

$$1.3244 \leq \int_0^1 e^x dx \leq 2.1835$$