

Question 3

Use Simpson's Rule with $n = 4$ to approximate

$$\int_0^2 \sqrt{1+x^3} \, dx.$$

Question 3 – Solution

Step 1: Find the width and nodes.

$$h = \frac{2-0}{4} = 0.5, \quad x_i = 0, 0.5, 1, 1.5, 2.$$

Step 2: Evaluate $f(x) = \sqrt{1+x^3}$.

$$\begin{aligned} f(0) &= 1, & f(0.5) &= \sqrt{1.125}, & f(1) &= \sqrt{2}, \\ f(1.5) &= \sqrt{4.375}, & f(2) &= 3. \end{aligned}$$

Step 3: Apply Simpson's Rule.

$$\begin{aligned} S_4 &= \frac{h}{3} [f(0) + 4f(0.5) + 2f(1) + 4f(1.5) + f(2)] \\ &= \frac{0.5}{3} [1 + 4\sqrt{1.125} + 2\sqrt{2} + 4\sqrt{4.375} + 3] \\ &\approx 3.23961135. \end{aligned}$$

$$\boxed{S_4 \approx 3.2396}$$