

Question 1

Evaluate by tabular integration:

$$\int x^3 e^{2x} dx.$$

Question 1 – Solution

We use tabular integration because repeatedly differentiating the polynomial x^3 eventually gives zero. Differentiate in the D column and integrate in the I column. The signs alternate, beginning with a plus sign.

Sign	D	I
+	x^3	$\frac{e^{2x}}{2}$
–	$3x^2$	$\frac{e^{2x}}{4}$
+	$6x$	$\frac{e^{2x}}{8}$
–	6	$\frac{e^{2x}}{16}$
	0	

For example, each entry in the integration column is obtained using

$$\int e^{2x} dx = \frac{1}{2}e^{2x},$$

so every step down that column introduces another factor of $\frac{1}{2}$.

Now multiply across each row, pairing each entry in the derivative column with the entry in the integration column on the same row, and use the indicated signs:

$$\begin{aligned} \int x^3 e^{2x} dx &= x^3 \left(\frac{e^{2x}}{2} \right) - 3x^2 \left(\frac{e^{2x}}{4} \right) + 6x \left(\frac{e^{2x}}{8} \right) - 6 \left(\frac{e^{2x}}{16} \right) + C \\ &= \frac{x^3 e^{2x}}{2} - \frac{3x^2 e^{2x}}{4} + \frac{6x e^{2x}}{8} - \frac{6e^{2x}}{16} + C \\ &= e^{2x} \left(\frac{x^3}{2} - \frac{3x^2}{4} + \frac{3x}{4} - \frac{3}{8} \right) + C. \end{aligned}$$

Therefore,

$$\boxed{\int x^3 e^{2x} dx = e^{2x} \left(\frac{x^3}{2} - \frac{3x^2}{4} + \frac{3x}{4} - \frac{3}{8} \right) + C}$$

As a check, differentiating the answer gives

$$\frac{d}{dx} \left[e^{2x} \left(\frac{x^3}{2} - \frac{3x^2}{4} + \frac{3x}{4} - \frac{3}{8} \right) \right] = x^3 e^{2x},$$

which is the original integrand.