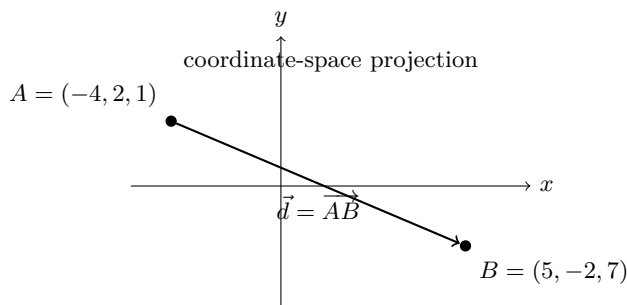


Question 1

A rescue drone travels from $A = (-4, 2, 1)$ to $B = (5, -2, 7)$. Find the displacement vector, its magnitude, and the corresponding unit direction vector.

Question 1 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 Subtract initial coordinates from terminal coordinates: $\vec{d} = B - A = \langle 9, -4, 6 \rangle$.

Step 2 Compute its length: $\|\vec{d}\| = \sqrt{9^2 + (-4)^2 + 6^2} = \sqrt{133}$.

Step 3 Normalize: $\hat{d} = \vec{d}/\|\vec{d}\| = \frac{1}{\sqrt{133}} \langle 9, -4, 6 \rangle$. The displacement magnitude is $\sqrt{133}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

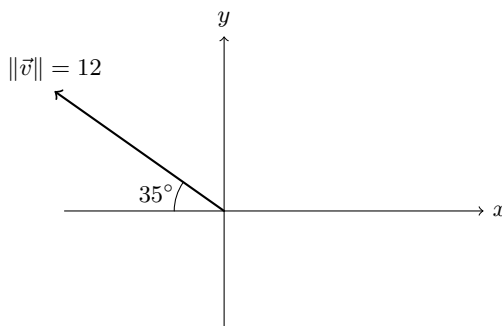
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 2

A vector in the plane has magnitude 12, points into Quadrant II, and makes an angle of 35° with the negative x -axis. Write it in component form.

Question 2 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 Its standard-position angle is $180^\circ - 35^\circ = 145^\circ$.

Step 2 Use $\vec{v} = \|\vec{v}\|\langle \cos \theta, \sin \theta \rangle$.

Step 3 Thus $\boxed{\vec{v} = 12\langle -\cos 35^\circ, \sin 35^\circ \rangle}$. The signs agree with Quadrant II.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

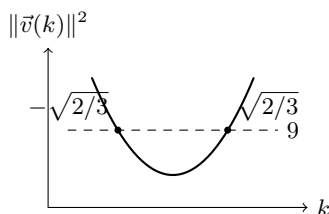
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 3

Find all real k for which $\vec{v} = \langle k, 2k - 1, k + 2 \rangle$ has magnitude 3.

Question 3 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 Square the magnitude equation: $k^2 + (2k - 1)^2 + (k + 2)^2 = 9$.

Step 2 Expanding gives $6k^2 + 5 = 9$, so $3k^2 = 2$.

Step 3 Therefore $\boxed{k = \pm\sqrt{2/3}}$; substitution verifies both values.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

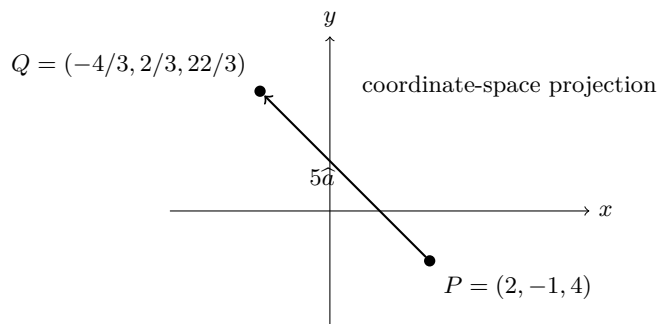
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 4

The point $P = (2, -1, 4)$ is translated five units in the direction of $\vec{a} = \langle -2, 1, 2 \rangle$. Find the image point Q .

Question 4 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 First normalize: $\|\vec{a}\| = 3$, so $\hat{a} = \frac{1}{3}\langle -2, 1, 2 \rangle$.

Step 2 A five-unit translation has vector $5\hat{a} = \langle -10/3, 5/3, 10/3 \rangle$.

Step 3 Add it to P : $Q = P + 5\hat{a} = \left(-\frac{4}{3}, \frac{2}{3}, \frac{22}{3} \right)$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

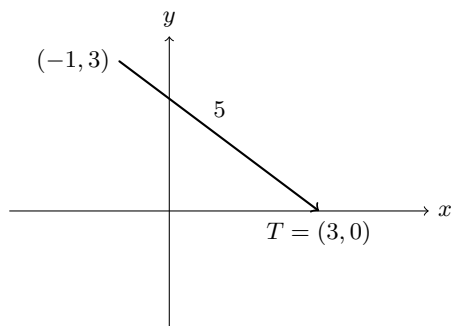
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 5

A vector $\vec{u}$ begins at $(-1, 3)$ and ends somewhere on the positive x -axis. If $\|\vec{u}\| = 5$, find every possible terminal point.

Question 5 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 Write the terminal point as $T = (t, 0)$ with $t > 0$. Then $\vec{u} = \langle t + 1, -3 \rangle$.

Step 2 The length condition gives $(t + 1)^2 + 9 = 25$, hence $t + 1 = \pm 4$.

Step 3 The candidates are $t = 3, -5$; positivity leaves $\boxed{T = (3, 0)}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

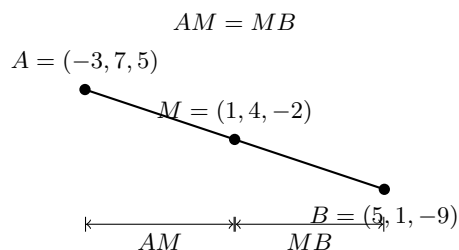
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 6

Show that $M = (1, 4, -2)$ is the midpoint of the segment joining $A = (-3, 7, 5)$ and an unknown point B , and determine B .

Question 6 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 The midpoint equation is $M = (A + B)/2$, so $B = 2M - A$.

Step 2 Compute $2M = \langle 2, 8, -4 \rangle$ and subtract A .

Step 3 Thus $B = \boxed{(5, 1, -9)}$. Averaging A and B returns $(1, 4, -2)$, which verifies the claim.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

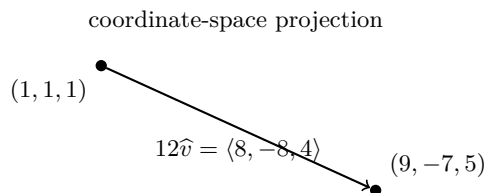
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 7

A particle moves from $(1, 1, 1)$ in the direction $\langle 2, -2, 1 \rangle$. At what point has it traveled exactly 12 units?

Question 7 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 The direction magnitude is $\sqrt{4 + 4 + 1} = 3$, so its unit vector is $\frac{1}{3}\langle 2, -2, 1 \rangle$.

Step 2 A 12-unit displacement is $12\hat{v} = \langle 8, -8, 4 \rangle$.

Step 3 Adding the starting point gives $(9, -7, 5)$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

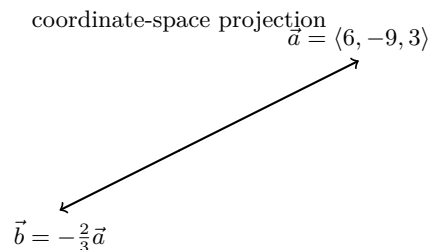
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 8

Determine whether $\vec{a} = \langle 6, -9, 3 \rangle$ and $\vec{b} = \langle -4, 6, -2 \rangle$ represent the same direction, opposite directions, or neither. Explain.

Question 8 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 Compare corresponding nonzero components: $-4/6 = 6/(-9) = (-2)/3 = -2/3$.

Step 2 Thus $\vec{b} = (-2/3)\vec{a}$.

Step 3 Because the scalar is negative, the vectors are parallel but point in opposite directions.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

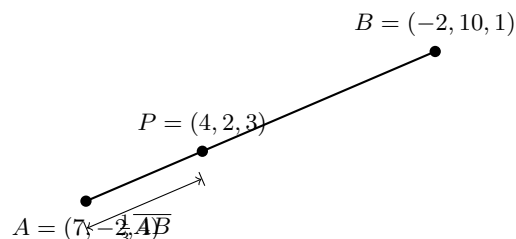
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 9

Find the point one-third of the way from $A = (7, -2, 4)$ to $B = (-2, 10, 1)$.

Question 9 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 The displacement from A to B is $B - A = \langle -9, 12, -3 \rangle$.

Step 2 One-third of this displacement is $\langle -3, 4, -1 \rangle$.

Step 3 Starting at A gives $A + \frac{1}{3}(B - A) = \boxed{(4, 2, 3)}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

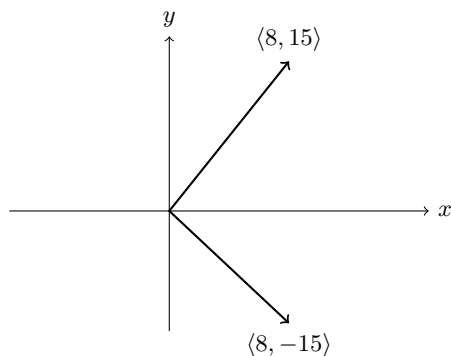
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 10

A velocity vector has horizontal component 8 and total speed 17. Find all possible vertical components and interpret the two answers.

Question 10 – Solution

Strategy Identify the initial and terminal information first. Build the vector by subtracting coordinates, then use the magnitude formula or normalize only after the components are correct.



Step 1 Let the vertical component be v . The speed condition is $\sqrt{8^2 + v^2} = 17$.

Step 2 Squaring yields $v^2 = 289 - 64 = 225$, so $v = \pm 15$.

Step 3 The velocities are $\boxed{\langle 8, 15 \rangle}$ and $\boxed{\langle 8, -15 \rangle}$, representing upward and downward motion with the same speed.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

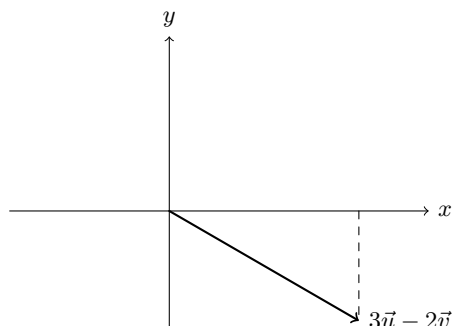
Key idea A vector separates two ideas: its components describe displacement, while its magnitude and unit vector describe size and direction. Always check signs and confirm that a unit vector has magnitude 1.

Question 1

Given $\vec{u} = \langle 2, -1, 3 \rangle$ and $\vec{v} = \langle -4, 5, 1 \rangle$, find $3\vec{u} - 2\vec{v}$ and its magnitude.

Question 1 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 Scale componentwise: $3\vec{u} = \langle 6, -3, 9 \rangle$ and $2\vec{v} = \langle -8, 10, 2 \rangle$.

Step 2 Subtract to get $3\vec{u} - 2\vec{v} = \langle 14, -13, 7 \rangle$.

Step 3 Its magnitude is $\sqrt{196 + 169 + 49} = \boxed{\sqrt{414} = 3\sqrt{46}}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

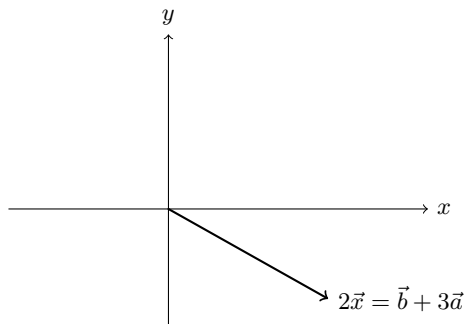
Key idea Vector equations behave like ordinary algebra provided that every operation is performed component by component. A final component check is often the fastest verification.

Question 2

Solve the vector equation $2\vec{x} - 3\vec{a} = \vec{b}$ when $\vec{a} = \langle 1, -2, 4 \rangle$ and $\vec{b} = \langle 5, 0, -6 \rangle$.

Question 2 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 Isolate the unknown: $2\vec{x} = \vec{b} + 3\vec{a}$.

Step 2 Compute $\vec{b} + 3\vec{a} = \langle 5, 0, -6 \rangle + \langle 3, -6, 12 \rangle = \langle 8, -6, 6 \rangle$.

Step 3 Divide by 2: $\boxed{\vec{x} = \langle 4, -3, 3 \rangle}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

Key idea Vector equations behave like ordinary algebra provided that every operation is performed component by component. A final component check is often the fastest verification.

Question 3

For what value of t are $\langle t + 1, 2t - 3, 5 \rangle$ and $\langle 4, 3, 5 \rangle$ equal? If none exists, identify the obstruction.

Question 3 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.

$$\langle t + 1, 2t - 3, 5 \rangle = \langle 4, 3, 5 \rangle$$

Step 1 Equal vectors require equality of every component.

Step 2 The first component gives $t + 1 = 4$, so $t = 3$; the second gives $2t - 3 = 3$, also $t = 3$.

Step 3 The third components already agree, so the unique value is $\boxed{t = 3}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

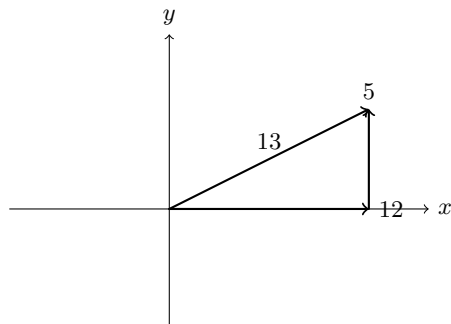
Key idea Vector equations behave like ordinary algebra provided that every operation is performed component by component. A final component check is often the fastest verification.

Question 4

A boat moves at 12 km/h due east relative to the water while a current flows at 5 km/h due north. Find the ground velocity, speed, and direction north of east.

Question 4 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 Add the velocity vectors: $\vec{v}_g = \langle 12, 0 \rangle + \langle 0, 5 \rangle = \langle 12, 5 \rangle$.

Step 2 The ground speed is $\sqrt{12^2 + 5^2} = 13$ km/h.

Step 3 The direction satisfies $\tan \theta = 5/12$, so $\theta = \tan^{-1}(5/12) \approx 22.6^\circ$ north of east.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

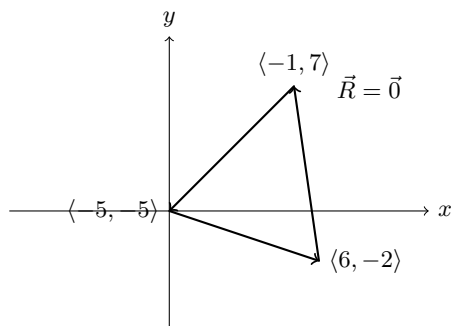
Key idea Vector equations behave like ordinary algebra provided that every operation is performed component by component. A final component check is often the fastest verification.

Question 5

Three forces $\langle 6, -2 \rangle$, $\langle -1, 7 \rangle$, and $\langle p, -5 \rangle$ act on an object. Find p so the horizontal component of the resultant vanishes, then find the resultant. Is it a nonzero vertical force?

Question 5 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 Add components: $\vec{R} = \langle 6 - 1 + p, -2 + 7 - 5 \rangle = \langle p + 5, 0 \rangle$.

Step 2 A vertical vector must have horizontal component zero, so $p + 5 = 0$ and $p = -5$.

Step 3 Here the vertical component is also zero, so the resultant is the zero vector: $\boxed{\vec{R} = \vec{0}}$. This is equilibrium. The zero vector has no direction, so no value of p produces a nonzero vertical resultant.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

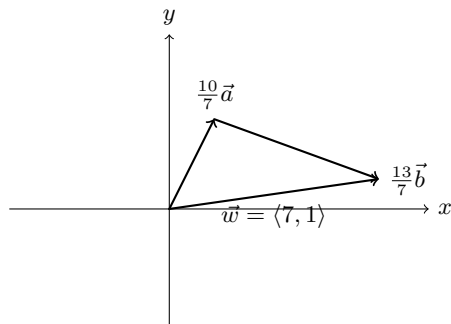
Key idea Vector equations behave like ordinary algebra provided that every operation is performed component by component. A final component check is often the fastest verification.

Question 6

Write $\vec{w} = \langle 7, 1 \rangle$ as a linear combination of $\vec{a} = \langle 1, 2 \rangle$ and $\vec{b} = \langle 3, -1 \rangle$.

Question 6 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 Let $\vec{w} = c\vec{a} + d\vec{b}$. Components give $c + 3d = 7$ and $2c - d = 1$.

Step 2 From the second equation $d = 2c - 1$. Substitute: $c + 3(2c - 1) = 7$, so $7c = 10$.

Step 3 Thus $c = 10/7$ and $d = 13/7$, giving $\vec{w} = \frac{10}{7}\vec{a} + \frac{13}{7}\vec{b}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

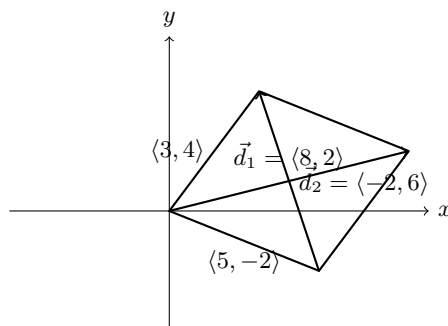
Key idea Vector equations behave like ordinary algebra provided that every operation is performed component by component. A final component check is often the fastest verification.

Question 7

The diagonals of a parallelogram are $\vec{d}_1 = \langle 8, 2 \rangle$ and $\vec{d}_2 = \langle -2, 6 \rangle$. Find possible adjacent side vectors.

Question 7 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 If adjacent sides are $\vec{a}, \vec{b}$, then $\vec{d}_1 = \vec{a} + \vec{b}$ and $\vec{d}_2 = \vec{a} - \vec{b}$.

Step 2 Add and divide by two: $\vec{a} = (\vec{d}_1 + \vec{d}_2)/2 = \langle 3, 4 \rangle$.

Step 3 Subtract and divide by two: $\vec{b} = (\vec{d}_1 - \vec{d}_2)/2 = \boxed{\langle 5, -2 \rangle}$. Thus one ordered pair is $\boxed{\langle 3, 4 \rangle, \langle 5, -2 \rangle}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

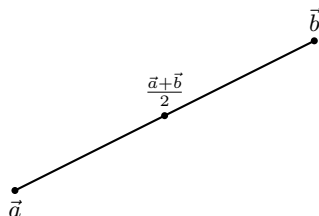
Key idea Vector equations behave like ordinary algebra provided that every operation is performed component by component. A final component check is often the fastest verification.

Question 8

Prove using components that the midpoint of points with position vectors $\vec{a}$ and $\vec{b}$ has position vector $(\vec{a} + \vec{b})/2$.

Question 8 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 Write $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$.

Step 2 Their average is $\frac{1}{2}(\vec{a} + \vec{b}) = \left\langle \frac{a_1+b_1}{2}, \frac{a_2+b_2}{2}, \frac{a_3+b_3}{2} \right\rangle$.

Step 3 Each coordinate is the ordinary midpoint of the corresponding coordinates, so this is exactly the midpoint position vector. Proved

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

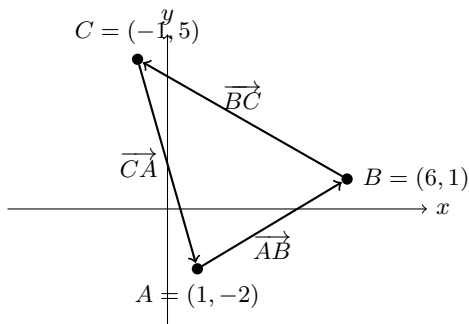
Key idea Vector equations behave like ordinary algebra provided that every operation is performed component by component. A final component check is often the fastest verification.

Question 9

A triangle has vertices $A = (1, -2)$, $B = (6, 1)$, and $C = (-1, 5)$. Verify the closed-path identity $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$.

Question 9 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 Compute $\vec{AB} = \langle 5, 3 \rangle$, $\vec{BC} = \langle -7, 4 \rangle$, and $\vec{CA} = \langle 2, -7 \rangle$.

Step 2 Add the three: $\langle 5 - 7 + 2, 3 + 4 - 7 \rangle = \langle 0, 0 \rangle$.

Step 3 Therefore $\boxed{\vec{AB} + \vec{BC} + \vec{CA} = \vec{0}}$, as any closed displacement path must.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

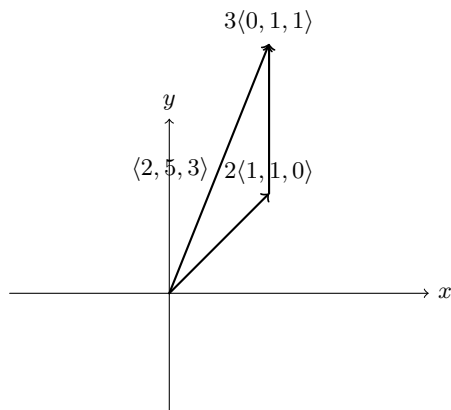
Key idea Vector equations behave like ordinary algebra provided that every operation is performed component by component. A final component check is often the fastest verification.

Question 10

Find constants a, b so that $a\langle 1, 1, 0 \rangle + b\langle 0, 1, 1 \rangle = \langle 2, 5, 3 \rangle$, or prove it is impossible.

Question 10 – Solution

Strategy Write every vector in matching component order. Perform scalar multiplication before addition or subtraction, then interpret the resulting components in the context of the problem.



Step 1 The linear combination is $\langle a, a + b, b \rangle$.

Step 2 The first and third components force $a = 2$ and $b = 3$.

Step 3 Then $a + b = 5$, matching the middle component. Hence $\boxed{a = 2, b = 3}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

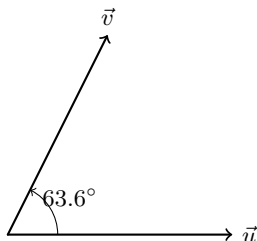
Key idea Vector equations behave like ordinary algebra provided that every operation is performed component by component. A final component check is often the fastest verification.

Question 1

Find the angle between $\vec{u} = \langle 1, 2, 2 \rangle$ and $\vec{v} = \langle 2, -1, 2 \rangle$.

Question 1 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 Compute $\vec{u} \cdot \vec{v} = 2 - 2 + 4 = 4$.

Step 2 Both magnitudes are 3, so $\cos \theta = 4/(3 \cdot 3) = 4/9$.

Step 3 Therefore $\theta = \cos^{-1}(4/9) \approx 63.6^\circ$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

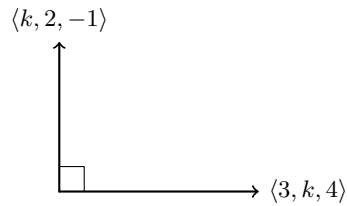
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 2

Find all k such that $\langle k, 2, -1 \rangle$ is perpendicular to $\langle 3, k, 4 \rangle$.

Question 2 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 Perpendicular vectors have zero dot product.

Step 2 Set $3k + 2k - 4 = 0$, which simplifies to $5k = 4$.

Step 3 Thus $k = 4/5$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

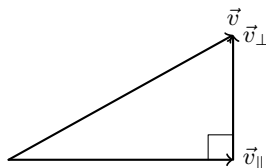
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 3

Decompose $\vec{v} = \langle 5, 1, -2 \rangle$ into components parallel and perpendicular to $\vec{u} = \langle 1, 2, 2 \rangle$.

Question 3 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 The parallel component is $\text{proj}_{\vec{u}} \vec{v} = \frac{\vec{v} \cdot \vec{u}}{\vec{u} \cdot \vec{u}} \vec{u}$. Here $\vec{v} \cdot \vec{u} = 3$ and $\vec{u} \cdot \vec{u} = 9$.

Step 2 So $\vec{v}_{\parallel} = \frac{1}{3} \langle 1, 2, 2 \rangle = \langle 1/3, 2/3, 2/3 \rangle$.

Step 3 Subtract: $\vec{v}_{\perp} = \vec{v} - \vec{v}_{\parallel} = \boxed{\langle 14/3, 1/3, -8/3 \rangle}$. Their dot product is zero.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

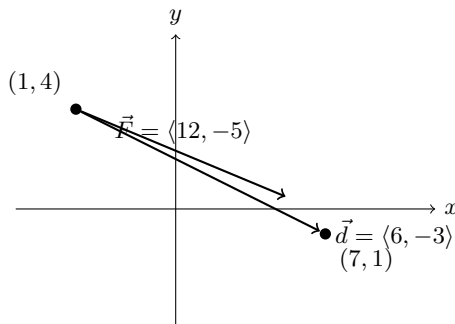
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 4

A force $\vec{F} = \langle 12, -5 \rangle$ moves an object from $(1, 4)$ to $(7, 1)$. Find the work done.

Question 4 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 The displacement is $\vec{d} = \langle 7 - 1, 1 - 4 \rangle = \langle 6, -3 \rangle$.

Step 2 Work is the dot product: $W = \vec{F} \cdot \vec{d}$.

Step 3 Thus $W = 12(6) + (-5)(-3) = 72 + 15 = \boxed{87}$ force-distance units.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

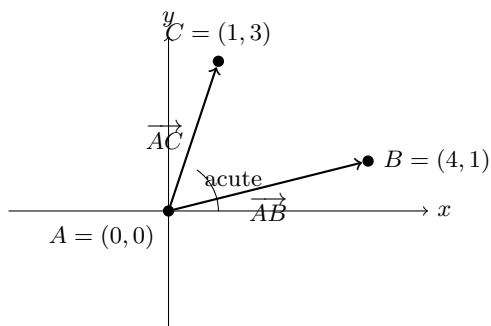
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 5

Determine whether the triangle with vertices $A = (0, 0)$, $B = (4, 1)$, and $C = (1, 3)$ has an acute, right, or obtuse angle at A .

Question 5 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 The sides from A are $\vec{AB} = \langle 4, 1 \rangle$ and $\vec{AC} = \langle 1, 3 \rangle$.

Step 2 Their dot product is $4(1) + 1(3) = 7 > 0$.

Step 3 A positive dot product means the included angle has positive cosine, so the angle at A is acute.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

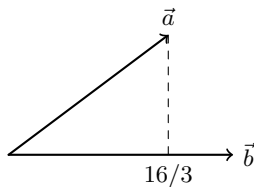
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 6

Find the scalar projection of $\vec{a} = \langle -2, 5, 4 \rangle$ onto $\vec{b} = \langle 1, 2, 2 \rangle$, and explain the sign.

Question 6 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 Compute $\vec{a} \cdot \vec{b} = -2 + 10 + 8 = 16$ and $\|\vec{b}\| = 3$.

Step 2 The scalar projection is $\text{comp}_{\vec{b}} \vec{a} = (\vec{a} \cdot \vec{b}) / \|\vec{b}\| = \boxed{16/3}$.

Step 3 It is positive because $\vec{a}$ has a net component in the same direction as $\vec{b}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

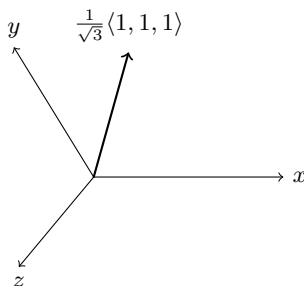
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 7

A unit vector $\vec{n}$ makes equal acute angles with the positive coordinate axes in $\mathbb{R}^3$. Find $\vec{n}$ and the common angle.

Question 7 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 Equal direction cosines imply $\vec{n} = \langle c, c, c \rangle$ with $c > 0$.

Step 2 The unit condition gives $3c^2 = 1$, so $c = 1/\sqrt{3}$.

Step 3 Hence $\vec{n} = \frac{1}{\sqrt{3}}\langle 1, 1, 1 \rangle$ and $\theta = \cos^{-1}(1/\sqrt{3})$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

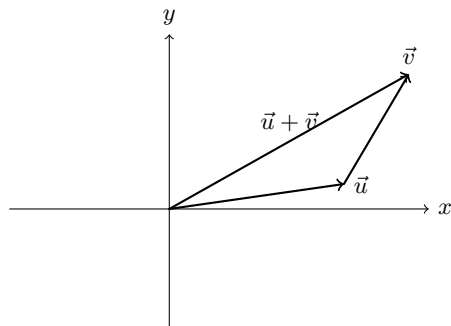
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 8

Prove the identity $\|\vec{u} + \vec{v}\|^2 = \|\vec{u}\|^2 + 2\vec{u} \cdot \vec{v} + \|\vec{v}\|^2$.

Question 8 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 Start with the squared norm: $\|\vec{u} + \vec{v}\|^2 = (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v})$.

Step 2 Distribute the dot product to obtain $\vec{u} \cdot \vec{u} + \vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{u} + \vec{v} \cdot \vec{v}$.

Step 3 Use symmetry and $\vec{u} \cdot \vec{u} = \|\vec{u}\|^2$ to get the stated identity. Proved

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

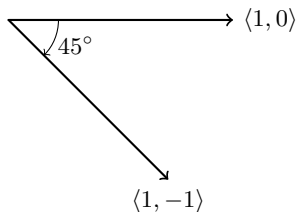
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 9

For which real t is the angle between $\langle 1, t \rangle$ and $\langle 1, -1 \rangle$ equal to 45° ?

Question 9 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 Use the cosine formula: $(1 - t)/(\sqrt{1 + t^2}\sqrt{2}) = \sqrt{2}/2$.

Step 2 This simplifies to $1 - t = \sqrt{1 + t^2}$. The right side is nonnegative, so $t \leq 1$. Squaring gives $1 - 2t + t^2 = 1 + t^2$, hence $t = 0$.

Step 3 It satisfies the original equation, so $\boxed{t = 0}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

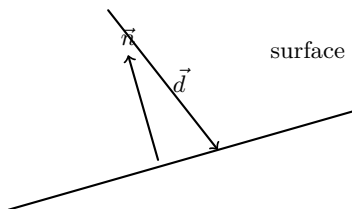
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 10

A spotlight beam points along $\vec{d} = \langle 2, -1, 2 \rangle$. A surface has unit normal $\vec{n} = \frac{1}{3}\langle 1, 2, 2 \rangle$. Find the magnitude of the component of $\vec{d}$ normal to the surface.

Question 10 – Solution

Strategy Compute the dot product carefully, then choose the formula that matches the question. A zero, positive, or negative dot product also gives immediate geometric information.



Step 1 Because $\vec{n}$ is a unit vector, the normal component magnitude is $|\vec{d} \cdot \vec{n}|$.

Step 2 Compute $\vec{d} \cdot \vec{n} = (2 - 2 + 4)/3 = 4/3$.

Step 3 Therefore the requested magnitude is $\boxed{4/3}$. The absolute value makes the result independent of the chosen normal orientation.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

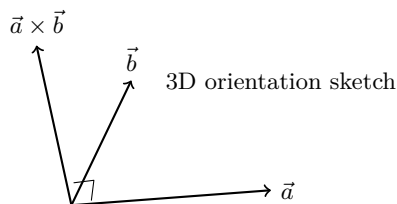
Key idea The dot product converts two vectors into a scalar measuring directional agreement: positive means generally aligned, zero means perpendicular, and negative means generally opposed.

Question 1

Compute $\vec{a} \times \vec{b}$ for $\vec{a} = \langle 1, 2, -1 \rangle$ and $\vec{b} = \langle 3, 0, 4 \rangle$, then verify it is perpendicular to both vectors.

Question 1 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Expand the determinant: $\vec{a} \times \vec{b} = \langle 2(4) - (-1)(0), (-1)(3) - 1(4), 1(0) - 2(3) \rangle = \langle 8, -7, -6 \rangle$.

Step 2 Check $(\vec{a} \times \vec{b}) \cdot \vec{a} = 8 - 14 + 6 = 0$.

Step 3 Also $(\vec{a} \times \vec{b}) \cdot \vec{b} = 24 - 24 = 0$, so $\boxed{\vec{a} \times \vec{b} = \langle 8, -7, -6 \rangle}$ is perpendicular to both.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

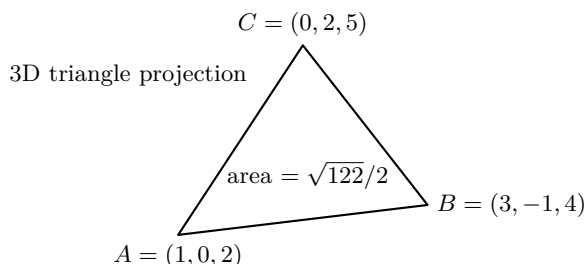
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 2

Find the area of the triangle with vertices $A = (1, 0, 2)$, $B = (3, -1, 4)$, and $C = (0, 2, 5)$.

Question 2 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Form two sides: $\overrightarrow{AB} = \langle 2, -1, 2 \rangle$ and $\overrightarrow{AC} = \langle -1, 2, 3 \rangle$.

Step 2 Their cross product is $\langle -7, -8, 3 \rangle$, whose magnitude is $\sqrt{49 + 64 + 9} = \sqrt{122}$.

Step 3 A triangle has half the parallelogram area, so $A = \sqrt{122}/2$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

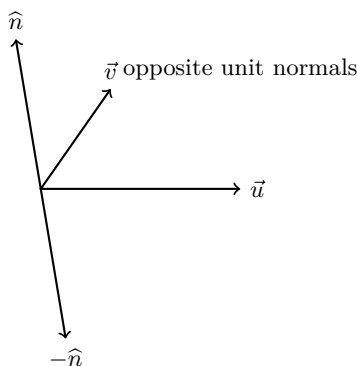
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 3

Find both unit vectors perpendicular to $\langle 2, 1, 0 \rangle$ and $\langle 1, -1, 3 \rangle$.

Question 3 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Take a cross product: $\langle 2, 1, 0 \rangle \times \langle 1, -1, 3 \rangle = \langle 3, -6, -3 \rangle = 3\langle 1, -2, -1 \rangle$.

Step 2 Its magnitude is $3\sqrt{6}$.

Step 3 Normalize and include both orientations: $\pm \frac{1}{\sqrt{6}} \langle 1, -2, -1 \rangle$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

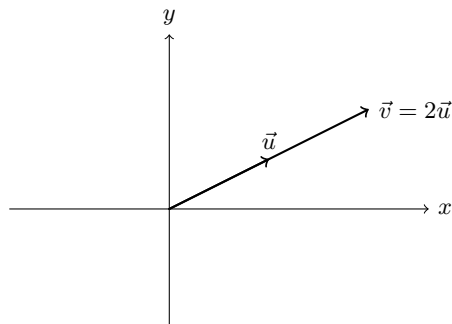
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 4

For what values of k are $\vec{u} = \langle 1, k, 2 \rangle$ and $\vec{v} = \langle 2, 2k, 4 \rangle$ parallel? What is their cross product then?

Question 4 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Observe directly that $\vec{v} = 2\vec{u}$ for every real k .

Step 2 Therefore the vectors are parallel for $\boxed{\text{all } k \in \mathbb{R}}$.

Step 3 The cross product of parallel vectors is the zero vector, so $\boxed{\vec{u} \times \vec{v} = \vec{0}}$ for every k .

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

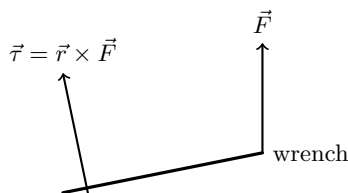
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 5

A wrench extends from the origin along $\vec{r} = \langle 0.30, 0.10, 0 \rangle$ meters. A force $\vec{F} = \langle 0, 40, -20 \rangle$ newtons is applied. Find the torque vector and its magnitude.

Question 5 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Torque is $\vec{\tau} = \vec{r} \times \vec{F}$.

Step 2 Compute $\vec{\tau} = \langle 0.10(-20), -0.30(-20), 0.30(40) \rangle = \langle -2, 6, 12 \rangle \text{ N}\cdot\text{m}$.

Step 3 Its magnitude is $\sqrt{4 + 36 + 144} = \boxed{2\sqrt{46} \text{ N}\cdot\text{m}}$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

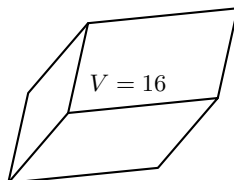
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 6

Find the volume of the parallelepiped determined by $\vec{a} = \langle 1, 2, 0 \rangle$, $\vec{b} = \langle 0, 1, 3 \rangle$, and $\vec{c} = \langle 2, -1, 1 \rangle$.

Question 6 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Volume is the absolute scalar triple product $|\vec{a} \cdot (\vec{b} \times \vec{c})|$.

Step 2 Compute $\vec{b} \times \vec{c} = \langle 4, 6, -2 \rangle$.

Step 3 Then $\vec{a} \cdot (\vec{b} \times \vec{c}) = 4 + 12 = 16$, so $\boxed{V = 16}$ cubic units.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

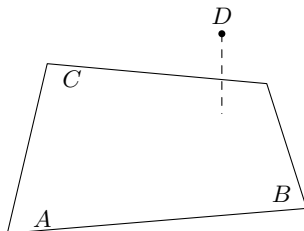
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 7

Determine whether the points $A = (1, 2, 0)$, $B = (3, 1, 4)$, $C = (-1, 5, 1)$, and $D = (5, -2, 7)$ are coplanar.

Question 7 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Use vectors from A : $\overrightarrow{AB} = \langle 2, -1, 4 \rangle$, $\overrightarrow{AC} = \langle -2, 3, 1 \rangle$, $\overrightarrow{AD} = \langle 4, -4, 7 \rangle$.

Step 2 Compute $\overrightarrow{AB} \times \overrightarrow{AC} = \langle -13, -10, 4 \rangle$.

Step 3 Dot with $\overrightarrow{AD}$: $-52 + 40 + 28 = 16 \neq 0$. Therefore the points are not coplanar.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

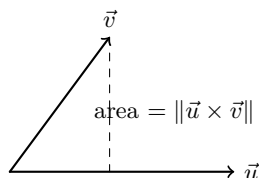
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 8

Prove the area identity $\|\vec{u} \times \vec{v}\|^2 = \|\vec{u}\|^2\|\vec{v}\|^2 - (\vec{u} \cdot \vec{v})^2$ using the angle between the vectors.

Question 8 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 If either vector is zero, both sides are zero. Otherwise, let θ be the angle between $\vec{u}$ and $\vec{v}$. Then $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\|\vec{v}\|\sin\theta$.

Step 2 Squaring and using $\sin^2\theta = 1 - \cos^2\theta$ gives $\|\vec{u}\|^2\|\vec{v}\|^2(1 - \cos^2\theta)$.

Step 3 Since $\vec{u} \cdot \vec{v} = \|\vec{u}\|\|\vec{v}\|\cos\theta$, substitution yields the claimed identity. Proved

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

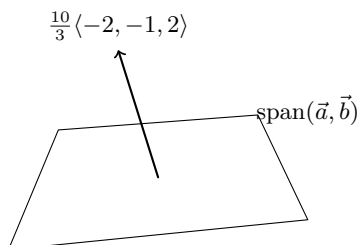
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 9

Find a vector of magnitude 10 perpendicular to the plane spanned by $\vec{a} = \langle 1, 0, 1 \rangle$ and $\vec{b} = \langle 0, 2, 1 \rangle$, with positive z -component.

Question 9 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 A normal direction is $\vec{a} \times \vec{b} = \langle -2, -1, 2 \rangle$.

Step 2 Its magnitude is 3, and its z -component is positive.

Step 3 Scale it to length 10: $\vec{n} = \frac{10}{3}\langle -2, -1, 2 \rangle$.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

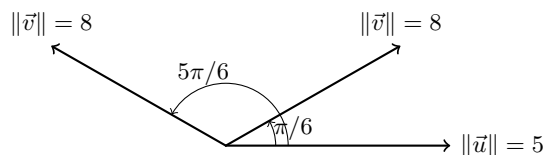
Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.

Question 10

Vectors $\vec{u}$ and $\vec{v}$ have lengths 5 and 8, and $\|\vec{u} \times \vec{v}\| = 20$. Find every possible angle between them in $[0, \pi]$.

Question 10 – Solution

Strategy Keep the component order and signs organized when expanding the determinant. After computing the cross product, use a dot-product check or the relevant magnitude formula to interpret it.



Step 1 Use $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\|\vec{v}\|\sin\theta$: $20 = 40\sin\theta$.

Step 2 Thus $\sin\theta = 1/2$.

Step 3 On $[0, \pi]$, the two solutions are $\theta = \pi/6$ or $5\pi/6$. The cross-product magnitude cannot distinguish acute from obtuse orientation.

Verification The result follows from the defining vector formulas used above, and each component, magnitude, or scalar condition has been checked against the information in the question.

Key idea The cross product produces a vector perpendicular to both inputs. Its direction follows the right-hand rule, while its magnitude records the area of the spanned parallelogram.