

Question 1

Find the volume of the solid obtained by rotating the region bounded by

$$y = \sqrt{x} \quad \text{and} \quad y = 0,$$

from $x = 0$ to $x = 4$, about the x -axis.



Rotate the shaded region about the x -axis.

Question 1 – Solution

1. Choose slices and bounds.

The rotation axis is horizontal, so use vertical slices of thickness dx . The stated interval is $0 \leq x \leq 4$. Each slice reaches the axis, producing a disk.

2. Measure the radii from the axis.

The distance from $y = 0$ to $y = \sqrt{x}$ is the outer radius. There is no central hole.

$$R(x) = \sqrt{x} - 0 = \sqrt{x}, \quad r(x) = 0.$$

3. Write the cross-sectional area and volume.

$$A(x) = \pi R(x)^2 - \pi r(x)^2 = \pi((\sqrt{x})^2 - 0^2) = \pi x.$$

$$V = \int_0^4 A(x) dx = \pi \int_0^4 x dx.$$

4. Integrate using the power rule.

$$\int x dx = \frac{x^{1+1}}{1+1} = \frac{x^2}{2}, \quad V = \pi \left[\frac{x^2}{2} \right]_0^4.$$

5. Apply the bounds and simplify.

$$V = \pi \left(\frac{4^2}{2} - \frac{0^2}{2} \right) = \pi \left(\frac{16}{2} \right) = \boxed{8\pi}.$$

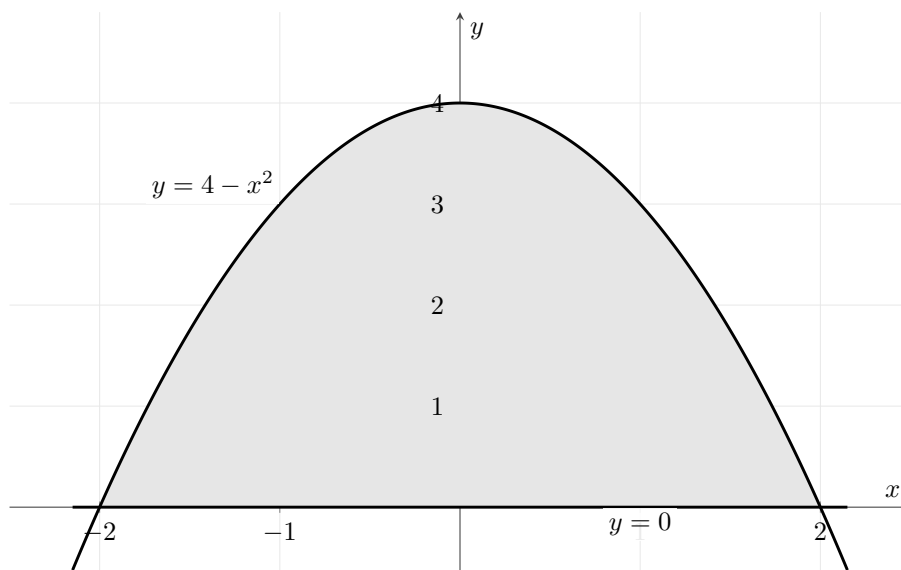
All volumes are in cubic units.

Question 2

Find the volume of the solid obtained by rotating the region bounded by

$$y = 4 - x^2 \quad \text{and} \quad y = 0,$$

from $x = -2$ to $x = 2$, about the x -axis.



Rotate the shaded region about the x -axis.

Question 2 – Solution**1. Choose slices and confirm the bounds.**

Use vertical slices, perpendicular to the x -axis. The curve meets $y = 0$ where

$$4 - x^2 = 0 \implies x = \pm 2.$$

2. Identify the radii.

On $[-2, 2]$, $4 - x^2 \geq 0$. The region touches the axis, so each cross-section is a disk:

$$R(x) = 4 - x^2, \quad r(x) = 0.$$

3. Set up the volume and expand the square.

$$V = \pi \int_{-2}^2 (R^2 - r^2) dx = \pi \int_{-2}^2 (4 - x^2)^2 dx.$$
$$(4 - x^2)^2 = 16 - 8x^2 + x^4.$$

4. Use symmetry and integrate.

The integrand is even, so double the integral over $[0, 2]$:

$$V = 2\pi \int_0^2 (16 - 8x^2 + x^4) dx = 2\pi \left[16x - \frac{8x^3}{3} + \frac{x^5}{5} \right]_0^2.$$

5. Substitute and combine the fractions.

$$V = 2\pi \left(32 - \frac{64}{3} + \frac{32}{5} \right)$$
$$= 2\pi \left(\frac{480 - 320 + 96}{15} \right) = \boxed{\frac{512\pi}{15}}.$$

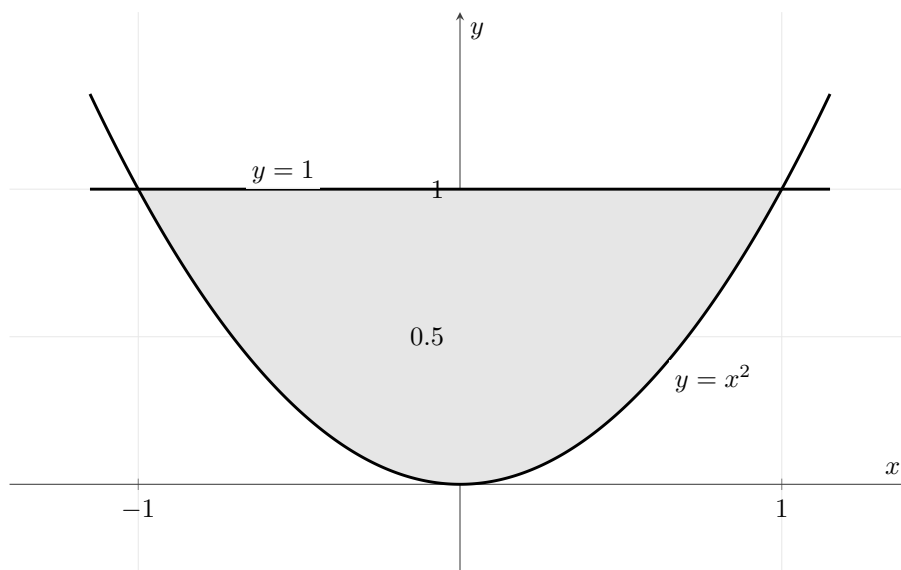
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Question 3

Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2 \quad \text{and} \quad y = 1,$$

from $x = -1$ to $x = 1$, about the x -axis.



Rotate the shaded region about the x -axis.

Question 3 – Solution

1. Find the bounds and choose slices.

The intersections satisfy $x^2 = 1$, so $x = -1$ and $x = 1$. Use vertical slices of thickness dx because the rotation axis is horizontal.

2. Identify outer and inner radii.

On $[-1, 1]$, $0 \leq x^2 \leq 1$. Measured from the x -axis,

$$R(x) = 1, \quad r(x) = x^2.$$

The missing inner disk must be subtracted from the outer disk.

3. Set up the washer integral.

$$A(x) = \pi(R(x)^2 - r(x)^2) = \pi(1^2 - (x^2)^2) = \pi(1 - x^4).$$

$$V = \pi \int_{-1}^1 (1 - x^4) dx.$$

4. Integrate term by term.

$$\int 1 dx = x, \quad \int x^4 dx = \frac{x^5}{5}.$$

$$V = \pi \left[x - \frac{x^5}{5} \right]_{-1}^1.$$

5. Evaluate both endpoints.

$$\begin{aligned} V &= \pi \left[\left(1 - \frac{1}{5} \right) - \left(-1 + \frac{1}{5} \right) \right] \\ &= \pi \left(\frac{4}{5} + \frac{4}{5} \right) = \boxed{\frac{8\pi}{5}}. \end{aligned}$$

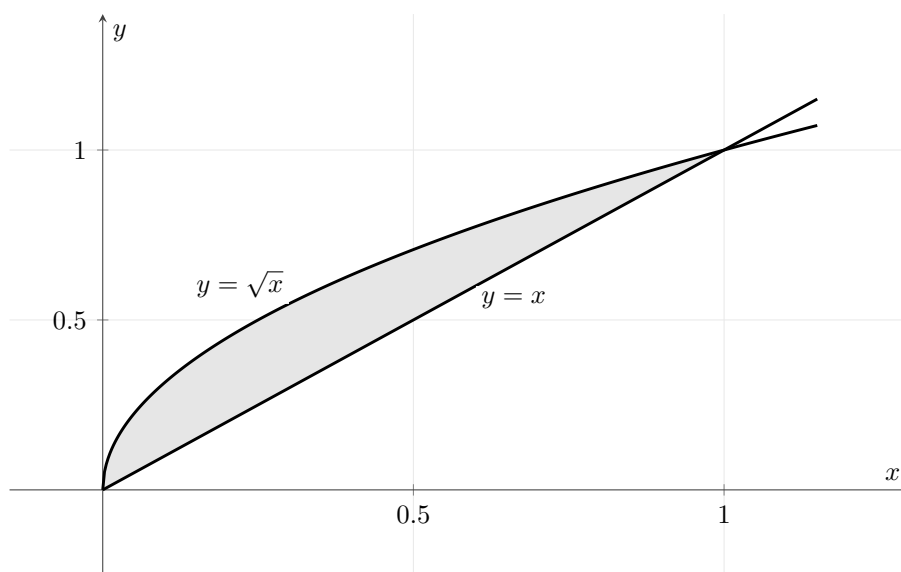
All volumes are in cubic units.

Question 4

Find the volume of the solid obtained by rotating the region bounded by

$$y = \sqrt{x} \quad \text{and} \quad y = x,$$

from $x = 0$ to $x = 1$, about the x -axis.



Rotate the shaded region about the x -axis.

Question 4 – Solution

1. Find the bounds and choose slices.

The curves meet where $\sqrt{x} = x$. With $x \geq 0$, squaring gives $x = x^2$, so $x = 0$ or $x = 1$. Use vertical slices for rotation about the x -axis.

2. Measure the radii.

For $0 \leq x \leq 1$, $\sqrt{x} \geq x \geq 0$. Thus the square-root curve is farther from the axis:

$$R(x) = \sqrt{x}, \quad r(x) = x.$$

3. Set up the washer integral.

Subtract the squared radii, then multiply by π :

$$V = \pi \int_0^1 (R^2 - r^2) dx = \pi \int_0^1 ((\sqrt{x})^2 - x^2) dx = \pi \int_0^1 (x - x^2) dx.$$

4. Find the antiderivative.

$$\int x dx = \frac{x^2}{2}, \quad \int x^2 dx = \frac{x^3}{3}.$$

$$V = \pi \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1.$$

5. Apply the bounds and simplify.

$$V = \pi \left[\left(\frac{1}{2} - \frac{1}{3} \right) - 0 \right] = \pi \left(\frac{3-2}{6} \right) = \boxed{\frac{\pi}{6}}.$$

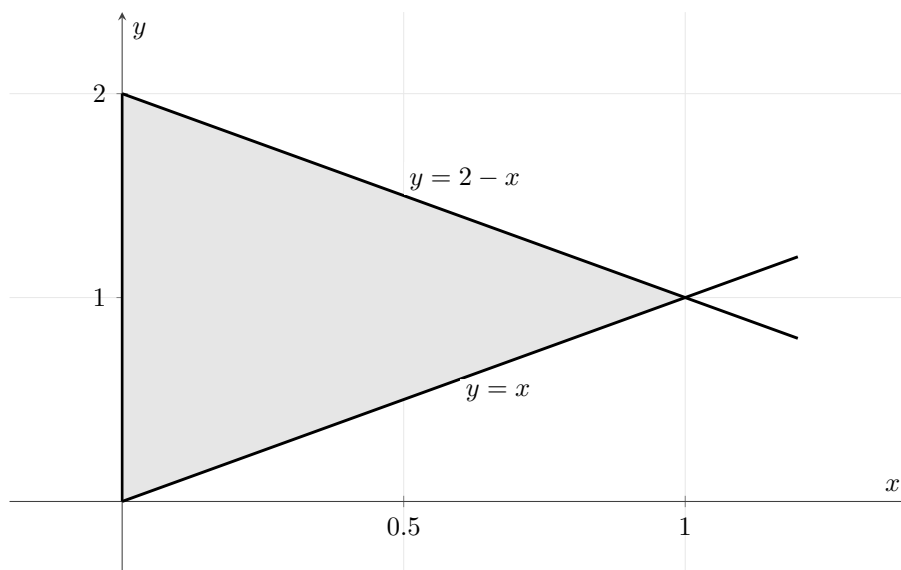
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Question 5

Find the volume of the solid obtained by rotating the region bounded by

$$y = 2 - x \quad \text{and} \quad y = x,$$

from $x = 0$ to $x = 1$, about the x -axis.



Rotate the shaded region about the x -axis.

Question 5 – Solution

1. Identify the bounds and slice direction.

The given left boundary is $x = 0$. The lines meet where $2 - x = x$, giving $x = 1$. Use vertical slices, perpendicular to the x -axis.

2. Measure the radii from the x -axis.

On $[0, 1]$, $2 - x \geq x \geq 0$, so

$$R(x) = 2 - x, \quad r(x) = x.$$

The rotated slice is a washer with area $\pi(R^2 - r^2)$.

3. Set up and simplify the volume integral.

$$\begin{aligned} V &= \pi \int_0^1 [(2 - x)^2 - x^2] dx. \\ (2 - x)^2 - x^2 &= (4 - 4x + x^2) - x^2 = 4 - 4x. \end{aligned}$$

4. Integrate term by term.

$$\begin{aligned} \int 4 dx &= 4x, & \int 4x dx &= 4 \frac{x^2}{2} = 2x^2. \\ V &= \pi [4x - 2x^2]_0^1. \end{aligned}$$

5. Apply the bounds.

$$V = \pi [(4(1) - 2(1)^2) - (4(0) - 2(0)^2)] = \pi(4 - 2) = \boxed{2\pi}.$$

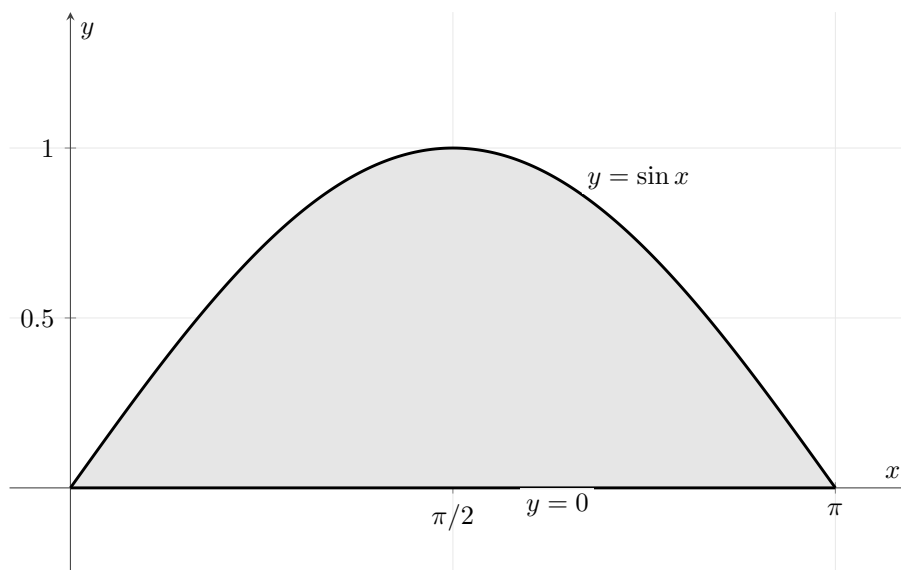
All volumes are in cubic units.

Question 6

Find the volume of the solid obtained by rotating the region bounded by

$$y = \sin x \quad \text{and} \quad y = 0,$$

from $x = 0$ to $x = \pi$, about the x -axis.



Rotate the shaded region about the x -axis.

Question 6 – Solution

1. Choose slices and identify the bounds.

Use vertical slices over $0 \leq x \leq \pi$. Since $\sin x \geq 0$ on this interval and the region touches the x -axis, rotation gives disks.

2. Identify radii and set up the volume.

$$R(x) = \sin x, \quad r(x) = 0.$$
$$V = \pi \int_0^\pi ((\sin x)^2 - 0^2) dx = \pi \int_0^\pi \sin^2 x \, dx.$$

3. Use the power-reduction identity.

$$\sin^2 x = \frac{1 - \cos(2x)}{2} \implies V = \frac{\pi}{2} \int_0^\pi (1 - \cos(2x)) \, dx.$$

4. Integrate, including the chain-rule factor.

Because $\frac{d}{dx} \sin(2x) = 2 \cos(2x)$,

$$\int \cos(2x) \, dx = \frac{\sin(2x)}{2}, \quad V = \frac{\pi}{2} \left[x - \frac{\sin(2x)}{2} \right]_0^\pi.$$

5. Evaluate at π and 0.

$$V = \frac{\pi}{2} \left[\left(\pi - \frac{\sin(2\pi)}{2} \right) - \left(0 - \frac{\sin 0}{2} \right) \right]$$
$$= \frac{\pi}{2} (\pi - 0) = \boxed{\frac{\pi^2}{2}}.$$

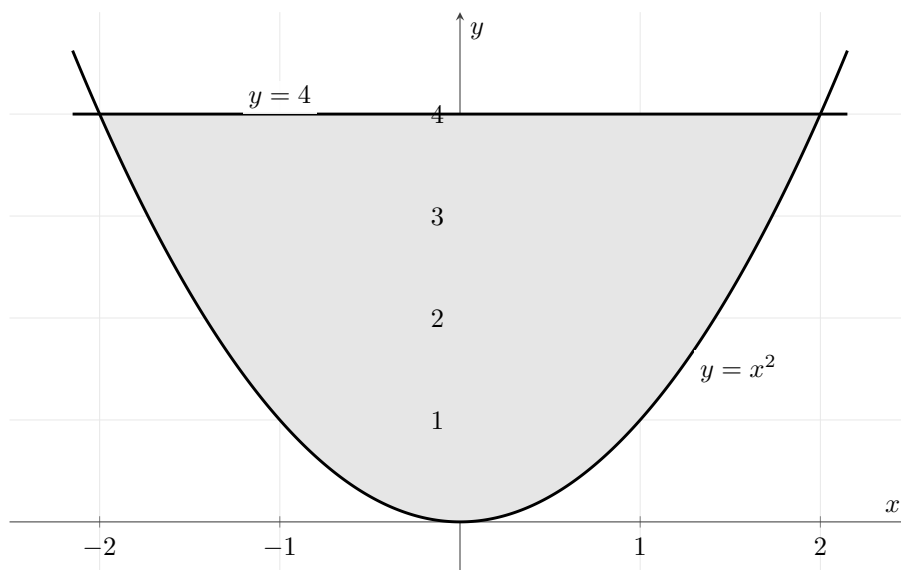
All volumes are in cubic units.

Question 7

Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2 \quad \text{and} \quad y = 4,$$

about the y -axis.



Rotate the shaded region about the y -axis.

Question 7 – Solution**1. Use horizontal slices and find the y -bounds.**

Rotation about the y -axis requires slices of thickness dy . The region lies above $y = x^2$ and below $y = 4$, so $0 \leq y \leq 4$.

2. Express the horizontal boundaries in terms of y .

$$y = x^2 \implies x = \pm\sqrt{y}.$$

At height y , the region extends from $-\sqrt{y}$ to $\sqrt{y}$ and crosses the rotation axis.

3. Identify the disk radius.

Rotating the entire horizontal segment gives one disk, not two separate disks. Its radius is the distance from the axis to either edge:

$$R(y) = \sqrt{y}, \quad r(y) = 0.$$

4. Set up the volume and integrate.

$$\begin{aligned} V &= \pi \int_0^4 (R^2 - r^2) dy = \pi \int_0^4 ((\sqrt{y})^2 - 0^2) dy = \pi \int_0^4 y dy. \\ \int y dy &= \frac{y^2}{2}. \end{aligned}$$

5. Apply the bounds.

$$V = \pi \left[\frac{y^2}{2} \right]_0^4 = \pi \left(\frac{16}{2} - 0 \right) = \boxed{8\pi}.$$

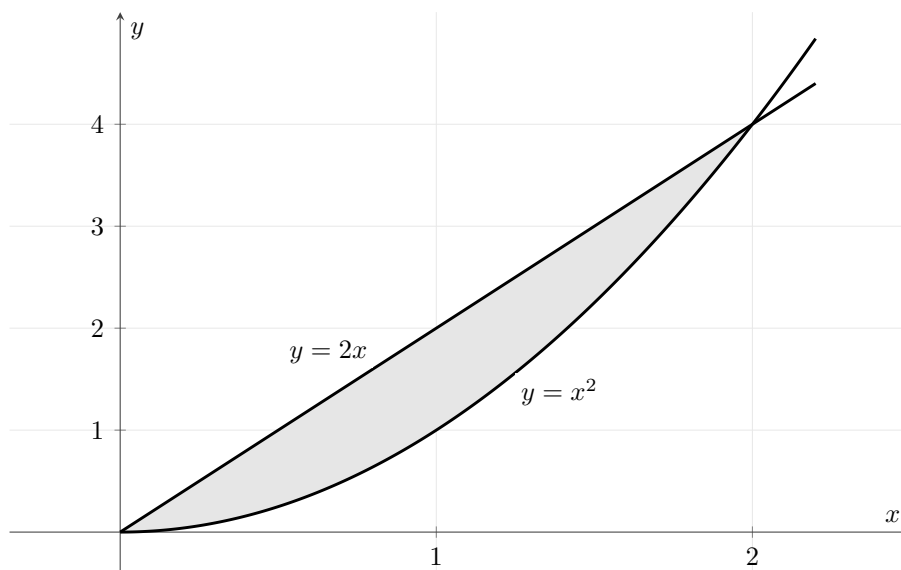
All volumes are in cubic units.

Question 8

Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2 \quad \text{and} \quad y = 2x,$$

about the y -axis.



Rotate the shaded region about the y -axis.

Question 8 – Solution

1. Find the intersections and y -bounds.

$$x^2 = 2x \implies x(x - 2) = 0 \implies x = 0, 2.$$

The intersection points are $(0, 0)$ and $(2, 4)$, so integrate from $y = 0$ to $y = 4$.

2. Use horizontal slices and identify radii.

Rewrite the curves as $x = \sqrt{y}$ and $x = y/2$. For $0 \leq y \leq 4$, $\sqrt{y} \geq y/2$. Distances from the y -axis give

$$R(y) = \sqrt{y}, \quad r(y) = \frac{y}{2}.$$

3. Set up the washer integral.

$$V = \pi \int_0^4 \left[(\sqrt{y})^2 - \left(\frac{y}{2}\right)^2 \right] dy = \pi \int_0^4 \left(y - \frac{y^2}{4} \right) dy.$$

4. Integrate term by term.

$$\int y \, dy = \frac{y^2}{2}, \quad \int \frac{y^2}{4} \, dy = \frac{1}{4} \frac{y^3}{3} = \frac{y^3}{12}.$$

$$V = \pi \left[\frac{y^2}{2} - \frac{y^3}{12} \right]_0^4.$$

5. Apply the bounds and simplify.

$$V = \pi \left(\frac{16}{2} - \frac{64}{12} - 0 \right) = \pi \left(8 - \frac{16}{3} \right) = \boxed{\frac{8\pi}{3}}.$$

All volumes are in cubic units.

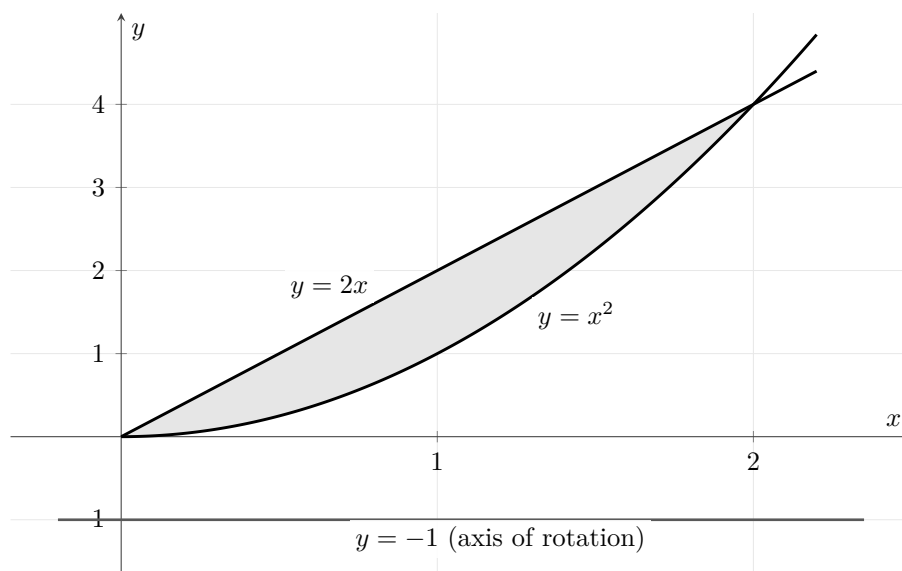
Question 9

Find the volume of the solid obtained by rotating the region bounded by

$$y = x^2 \quad \text{and} \quad y = 2x,$$

about the horizontal line

$$y = -1.$$



Rotate the shaded region about the line $y = -1$.

Question 9 – Solution**1. Find the bounds and choose vertical slices.**

The curves meet where $x^2 = 2x$, or $x(x - 2) = 0$. Thus $0 \leq x \leq 2$. The horizontal rotation axis $y = -1$ requires vertical slices of thickness dx .

2. Measure both radii from $y = -1$.

On $[0, 2]$, $2x \geq x^2$. The outer radius reaches the line, and the inner radius reaches the parabola:

$$R(x) = 2x - (-1) = 2x + 1, \quad r(x) = x^2 - (-1) = x^2 + 1.$$

3. Set up the washer integral and expand.

$$\begin{aligned} V &= \pi \int_0^2 [(2x + 1)^2 - (x^2 + 1)^2] dx. \\ (2x + 1)^2 - (x^2 + 1)^2 &= (4x^2 + 4x + 1) - (x^4 + 2x^2 + 1) \\ &= -x^4 + 2x^2 + 4x. \end{aligned}$$

4. Integrate each power.

$$V = \pi \left[-\frac{x^5}{5} + \frac{2x^3}{3} + 2x^2 \right]_0^2.$$

The coefficients follow from $\int x^n dx = x^{n+1}/(n+1)$.

5. Evaluate and use a common denominator.

$$\begin{aligned} V &= \pi \left(-\frac{32}{5} + \frac{16}{3} + 8 - 0 \right) \\ &= \pi \left(\frac{-96 + 80 + 120}{15} \right) = \boxed{\frac{104\pi}{15}}. \end{aligned}$$

All volumes are in cubic units.

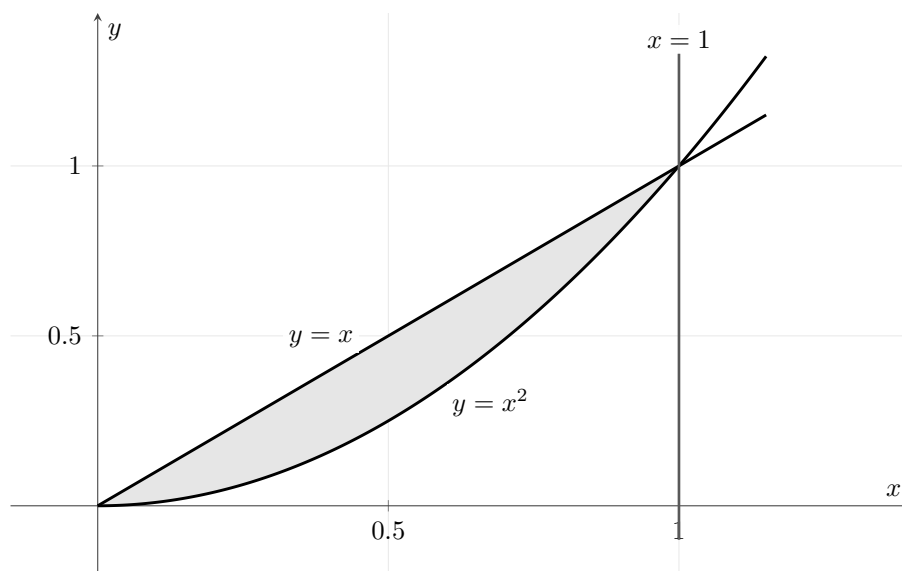
Question 10

Find the volume of the solid obtained by rotating the region bounded by

$$y = x \quad \text{and} \quad y = x^2,$$

about the vertical line

$$x = 1.$$



Rotate the shaded region about the line $x = 1$.

Question 10 – Solution**1. Find the bounds and choose horizontal slices.**

The intersections satisfy $x = x^2$, so $x = 0, 1$ and $y = 0, 1$. Rotation about the vertical line $x = 1$ requires horizontal slices of thickness dy .

2. Rewrite the curves and measure radii.

The boundaries are $x = y$ on the left and $x = \sqrt{y}$ on the right. The axis is to their right, so the farther, left boundary gives the outer radius:

$$R(y) = 1 - y, \quad r(y) = 1 - \sqrt{y}.$$

3. Set up the washer integral and expand.

$$\begin{aligned} V &= \pi \int_0^1 [(1 - y)^2 - (1 - \sqrt{y})^2] dy. \\ (1 - y)^2 - (1 - \sqrt{y})^2 &= (1 - 2y + y^2) - (1 - 2\sqrt{y} + y) \\ &= y^2 - 3y + 2y^{1/2}. \end{aligned}$$

4. Integrate using the power rule.

In particular, $\int 2y^{1/2} dy = 2 \cdot \frac{2}{3}y^{3/2} = \frac{4}{3}y^{3/2}$. Therefore,

$$V = \pi \left[\frac{y^3}{3} - \frac{3y^2}{2} + \frac{4}{3}y^{3/2} \right]_0^1.$$

5. Apply the bounds and simplify.

$$V = \pi \left(\frac{1}{3} - \frac{3}{2} + \frac{4}{3} - 0 \right) = \pi \left(\frac{2 - 9 + 8}{6} \right) = \boxed{\frac{\pi}{6}}.$$

All volumes are in cubic units.