

Question 1

Let:

$$f(x) = x^3 - 6x^2 + 9x + 2$$

Find the absolute maximum and minimum values of $f(x)$ on the closed interval $[0, 4]$.

Question 1 - Solution

We are given:

$$f(x) = x^3 - 6x^2 + 9x + 2$$

Step 1: Find critical points

$$f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3) = 0 \Rightarrow x = 1, 3$$

Step 2: Evaluate at endpoints and critical points in $[0, 4]$

$$f(0) = 2$$

$$f(1) = 6$$

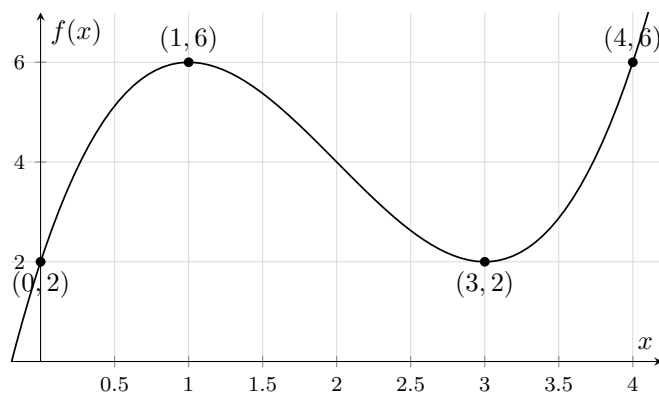
$$f(3) = 2$$

$$f(4) = 6$$

Absolute Maximum: $\boxed{6}$ at $x = 1$ and $x = 4$

Absolute Minimum: $\boxed{2}$ at $x = 0$ and $x = 3$

Graph of the function on $[0, 4]$



Question 2**Problem:**

Find the absolute maximum and minimum values of the function

$$f(x) = x^3 - 3x^2 + 1$$

on the interval $[0, 4]$.

- (a) Identify all critical points of $f(x)$ in the interval.
- (b) Evaluate $f(x)$ at the endpoints and critical points.
- (c) Determine the absolute maximum and minimum values.

Question 2 - Solution

We are given:

$$f(x) = x^3 - 3x^2 + 1$$

(a) **Find critical points:**

Take the derivative:

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

Set $f'(x) = 0$:

$$3x(x - 2) = 0 \Rightarrow x = 0, x = 2$$

Both are in the interval $[0, 4]$.

(b) **Evaluate function at critical points and endpoints:**

$$f(0) = 0^3 - 3(0)^2 + 1 = 1$$

$$f(2) = 8 - 12 + 1 = -3$$

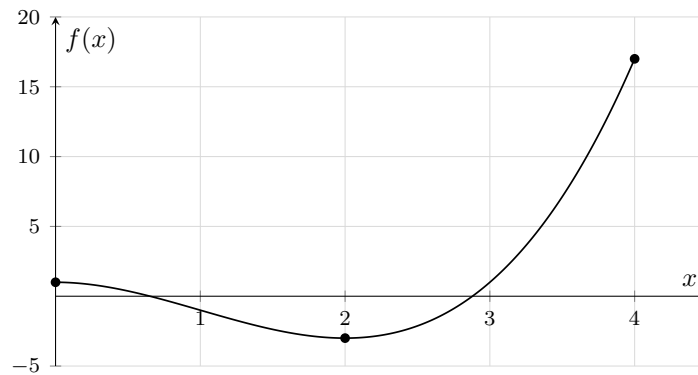
$$f(4) = 64 - 48 + 1 = 17$$

(c) **Absolute extrema:**

• Absolute minimum: $f(2) = -3$

• Absolute maximum: $f(4) = 17$

Graph of $f(x) = x^3 - 3x^2 + 1$ on $[0, 4]$:



Question 3**Problem:**

Find the absolute maximum and minimum values of the function

$$f(x) = \frac{x}{x^2 + 1}$$

on the interval $[-2, 2]$.

- (a) Find the critical points of $f(x)$ in the interval.
- (b) Evaluate $f(x)$ at the endpoints and at critical points.
- (c) Determine the absolute maximum and minimum values.

Question 3 - Solution

We are given:

$$f(x) = \frac{x}{x^2 + 1}$$

(a) Find critical points:

Use quotient rule:

$$f'(x) = \frac{(x^2 + 1)(1) - x(2x)}{(x^2 + 1)^2} = \frac{x^2 + 1 - 2x^2}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2}$$

Set $f'(x) = 0$:

$$\frac{1 - x^2}{(x^2 + 1)^2} = 0 \Rightarrow 1 - x^2 = 0 \Rightarrow x = \pm 1$$

So, critical points: $x = -1$, $x = 1$

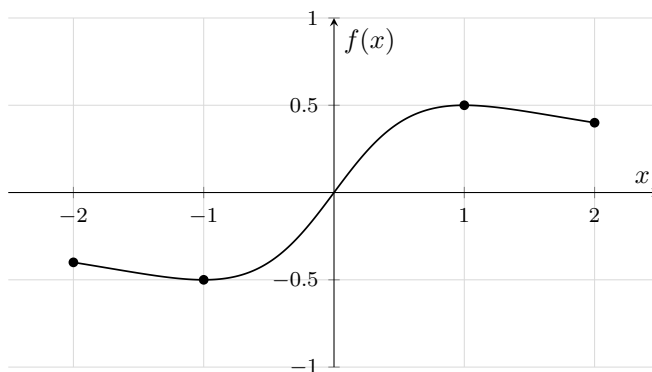
(b) Evaluate at endpoints and critical points:

$$f(-2) = \frac{-2}{4 + 1} = -\frac{2}{5}, \quad f(-1) = \frac{-1}{1 + 1} = -\frac{1}{2}$$
$$f(1) = \frac{1}{2}, \quad f(2) = \frac{2}{5}$$

(c) Conclusion:

- Absolute minimum: $f(-1) = -\frac{1}{2}$
- Absolute maximum: $f(1) = \frac{1}{2}$

Graph of $f(x) = \frac{x}{x^2 + 1}$ on $[-2, 2]$:



Question 4**Problem:**

Find the absolute maximum and minimum values of the function

$$f(x) = x^2 \ln(x)$$

on the interval $[1, 4]$.

- (a) Find the critical points of $f(x)$ in the interval.
- (b) Evaluate $f(x)$ at the endpoints and critical points.
- (c) Determine the absolute maximum and minimum values.

Question 4 - Solution

We are given:

$$f(x) = x^2 \ln(x)$$

(a) Find critical points:

Use product rule:

$$f'(x) = \frac{d}{dx}[x^2] \cdot \ln(x) + x^2 \cdot \frac{d}{dx}[\ln(x)] = 2x \ln(x) + x$$

Set derivative to zero:

$$f'(x) = 2x \ln(x) + x = x(2 \ln(x) + 1) = 0 \Rightarrow 2 \ln(x) + 1 = 0 \Rightarrow \ln(x) = -\frac{1}{2} \Rightarrow x = e^{-1/2}$$

Since $e^{-1/2} \approx 0.606 \notin [1, 4]$, there are no critical points in the interval.

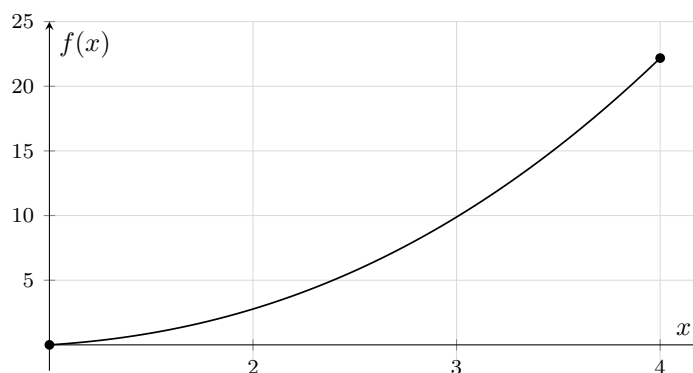
(b) Evaluate at endpoints:

$$f(1) = 1^2 \ln(1) = 0, \quad f(4) = 16 \ln(4) = 16 \cdot \ln(2^2) = 16 \cdot 2 \ln(2) \approx 16 \cdot 1.386 = 22.18$$

(c) Conclusion:

- Absolute minimum: $f(1) = 0$
- Absolute maximum: $f(4) \approx 22.18$

Graph of $f(x) = x^2 \ln(x)$ on $[1, 4]$:



Question 5**Problem:**

Find the absolute maximum and minimum values of the function

$$f(x) = x^3 - 3x + 1$$

on the interval $[-2, 2]$.

- (a) Find the critical points of $f(x)$ in the interval.
- (b) Evaluate $f(x)$ at the endpoints and critical points.
- (c) Identify the absolute maximum and minimum values.

Question 5 - Solution

We are given:

$$f(x) = x^3 - 3x + 1$$

(a) Find critical points:

$$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x - 1)(x + 1)$$

Set derivative to zero:

$$f'(x) = 0 \Rightarrow x = \pm 1$$

Critical points: $x = -1, x = 1$

(b) Evaluate function:

$$f(-2) = (-2)^3 - 3(-2) + 1 = -8 + 6 + 1 = -1$$

$$f(-1) = (-1)^3 - 3(-1) + 1 = -1 + 3 + 1 = 3$$

$$f(1) = 1 - 3 + 1 = -1$$

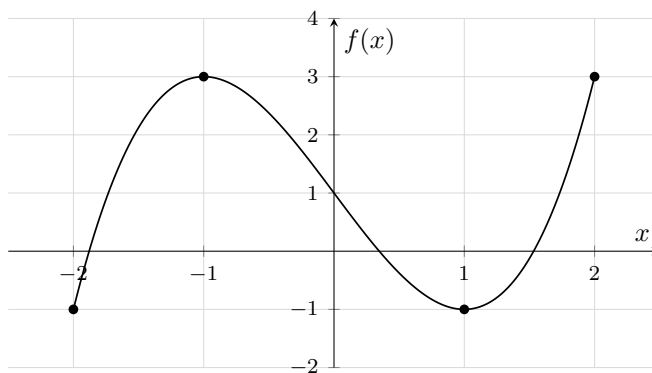
$$f(2) = 8 - 6 + 1 = 3$$

(c) Conclusion:

• Absolute maximum: $f(-1) = f(2) = 3$

• Absolute minimum: $f(-2) = f(1) = -1$

Graph of $f(x) = x^3 - 3x + 1$ on $[-2, 2]$:



Question 6**Problem:**

Let $f(x) = x^4 - 4x^2$.

- (a) Find the critical points of $f(x)$ in the interval $[-3, 3]$.
- (b) Evaluate the function at the critical points and endpoints.
- (c) Identify the absolute maximum and minimum values of $f(x)$ on the given interval.

Question 6 - Solution

We are given:

$$f(x) = x^4 - 4x^2$$

(a) Find critical points:

$$f'(x) = 4x^3 - 8x = 4x(x^2 - 2)$$

Set derivative to zero:

$$f'(x) = 0 \Rightarrow x = 0, \pm\sqrt{2}$$

(b) Evaluate at critical points and endpoints:

$$f(-3) = (-3)^4 - 4(-3)^2 = 81 - 36 = 45$$

$$f(-\sqrt{2}) = (\sqrt{2})^4 - 4(\sqrt{2})^2 = 4 - 8 = -4$$

$$f(0) = 0^4 - 4(0)^2 = 0$$

$$f(\sqrt{2}) = \text{same as } f(-\sqrt{2}) = -4$$

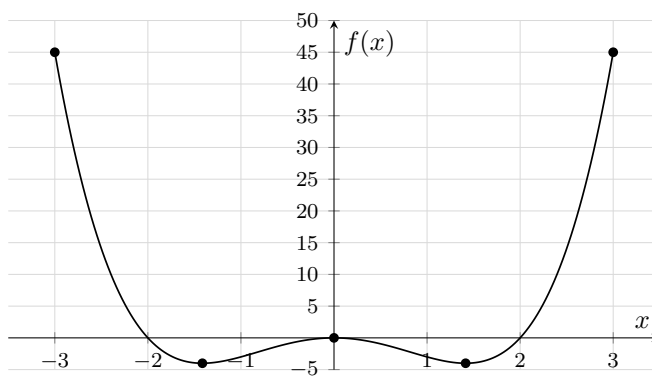
$$f(3) = 81 - 36 = 45$$

(c) Conclusion:

- Absolute maximum: $f(-3) = f(3) = 45$

- Absolute minimum: $f(\pm\sqrt{2}) = -4$

Graph of $f(x) = x^4 - 4x^2$ on $[-3, 3]$:



Question 7**Problem:**

A company models its daily profit (in hundreds of dollars) as a function of the number of units produced, using the function:

$$P(x) = -x^3 + 6x^2 + 15$$

where x is the number of units (in hundreds) produced per day, and $x \in [0, 5]$.

- (a) Find all critical points of $P(x)$ on the interval.
- (b) Determine the absolute maximum and minimum profit on the interval $[0, 5]$.

Question 7 - Solution

We are given:

$$P(x) = -x^3 + 6x^2 + 15$$

(a) Find critical points:

Differentiate:

$$P'(x) = -3x^2 + 12x = -3x(x - 4)$$

Set derivative equal to zero:

$$-3x(x - 4) = 0 \Rightarrow x = 0, x = 4$$

These are critical points in the interval $[0, 5]$.

(b) Evaluate endpoints and critical points:

$$P(0) = -0 + 0 + 15 = 15$$

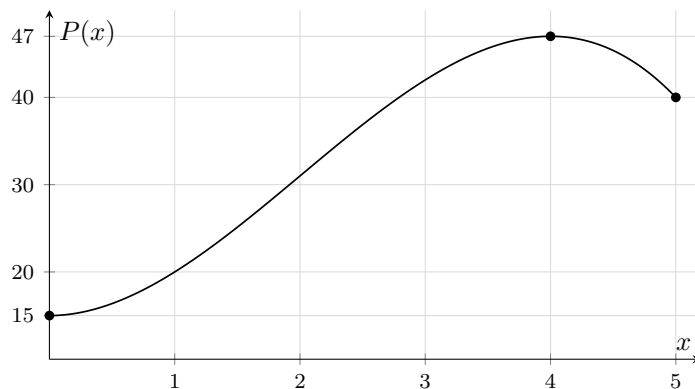
$$P(4) = -64 + 96 + 15 = 47$$

$$P(5) = -125 + 150 + 15 = 40$$

Conclusion:

- Absolute Maximum: $P(4) = 47$
- Absolute Minimum: $P(0) = 15$

Graph of $P(x) = -x^3 + 6x^2 + 15$ on $[0, 5]$:



Question 8**Problem:**

Let $f(x) = \frac{x}{x^2+1}$.

- (a) Find all critical points of $f(x)$ on the interval $[-3, 3]$.
- (b) Evaluate $f(x)$ at the critical points and endpoints.
- (c) Determine the absolute maximum and minimum values of $f(x)$ on the given interval.

Question 8 - Solution

We are given:

$$f(x) = \frac{x}{x^2 + 1}$$

(a) Find critical points:

Use the quotient rule:

$$f'(x) = \frac{(x^2 + 1)(1) - x(2x)}{(x^2 + 1)^2} = \frac{x^2 + 1 - 2x^2}{(x^2 + 1)^2} = \frac{-x^2 + 1}{(x^2 + 1)^2}$$

Set numerator to 0:

$$-x^2 + 1 = 0 \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

(b) Evaluate at endpoints and critical points:

$$f(-3) = \frac{-3}{9 + 1} = -\frac{3}{10}$$

$$f(-1) = \frac{-1}{1 + 1} = -\frac{1}{2}$$

$$f(1) = \frac{1}{1 + 1} = \frac{1}{2}$$

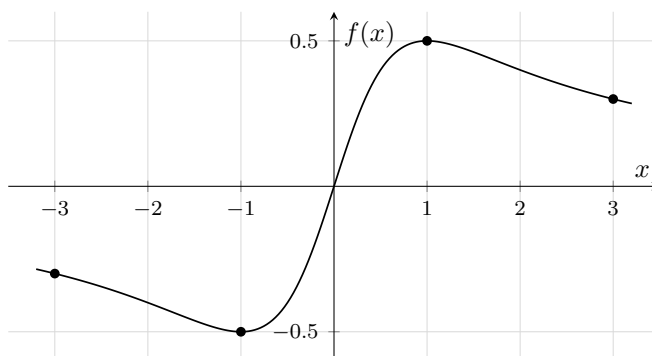
$$f(3) = \frac{3}{9 + 1} = \frac{3}{10}$$

(c) Conclusion:

• Absolute maximum: $f(1) = \frac{1}{2}$

• Absolute minimum: $f(-1) = -\frac{1}{2}$

Graph of $f(x) = \frac{x}{x^2 + 1}$ on $[-3, 3]$:



Question 9**Problem:**

Let $f(x) = x^3 - 3x^2 + 1$.

- (a) Find the critical points of $f(x)$ on the interval $[0, 3]$.
- (b) Evaluate the function at the critical points and endpoints.
- (c) Determine the absolute maximum and minimum values of $f(x)$ on the interval.

Question 9 - Solution

We are given:

$$f(x) = x^3 - 3x^2 + 1$$

(a) Find critical points:

$$f'(x) = 3x^2 - 6x = 3x(x - 2)$$

Set derivative to zero:

$$3x(x - 2) = 0 \Rightarrow x = 0, x = 2$$

(b) Evaluate $f(x)$ at critical points and endpoints:

$$f(0) = 0 - 0 + 1 = 1$$

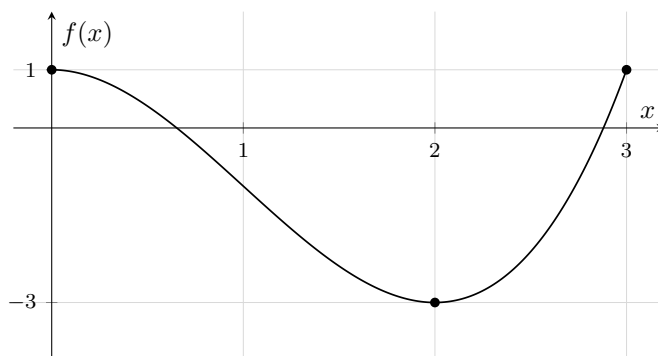
$$f(2) = 8 - 12 + 1 = -3$$

$$f(3) = 27 - 27 + 1 = 1$$

(c) Absolute extrema:

- Absolute minimum: $f(2) = -3$
- Absolute maximum: $f(0) = f(3) = 1$

Graph of $f(x) = x^3 - 3x^2 + 1$ on $[0, 3]$:



Question 10**Problem:**

Let $f(x) = \frac{x}{x^2+1}$.

- (a) Find the critical points of $f(x)$ on the interval $[-2, 2]$.
- (b) Evaluate the function at the critical points and endpoints.
- (c) Determine the absolute maximum and minimum values of $f(x)$ on the interval.

Question 10 - Solution

We are given:

$$f(x) = \frac{x}{x^2 + 1}$$

(a) Find critical points:

Use quotient rule:

$$f'(x) = \frac{(x^2 + 1)(1) - x(2x)}{(x^2 + 1)^2} = \frac{x^2 + 1 - 2x^2}{(x^2 + 1)^2} = \frac{1 - x^2}{(x^2 + 1)^2}$$

Set $f'(x) = 0$:

$$1 - x^2 = 0 \Rightarrow x = \pm 1$$

(b) Evaluate $f(x)$:

$$f(-2) = \frac{-2}{4+1} = -\frac{2}{5}, \quad f(-1) = \frac{-1}{1+1} = -\frac{1}{2}, \quad f(1) = \frac{1}{1+1} = \frac{1}{2}, \quad f(2) = \frac{2}{4+1} = \frac{2}{5}$$

(c) Absolute extrema:

• Absolute maximum: $f(1) = \frac{1}{2}$

• Absolute minimum: $f(-1) = -\frac{1}{2}$

Graph of $f(x) = \frac{x}{x^2+1}$ on $[-2, 2]$:

