

Question 1

A company's revenue (in dollars) from selling x items is given by:

$$R(x) = 50x - 0.2x^2$$

and the cost (in dollars) to produce x items is:

$$C(x) = 10x + 100$$

- (a) Find the profit function $P(x)$.
- (b) Find the number of items that maximizes profit.
- (c) What is the maximum profit?

Question 1 - Solution

(a) Profit is revenue minus cost:

$$P(x) = R(x) - C(x) = (50x - 0.2x^2) - (10x + 100) = 40x - 0.2x^2 - 100$$

(b) Maximize $P(x) = -0.2x^2 + 40x - 100$.

This is a downward-opening parabola. The maximum occurs at:

$$x = \frac{-b}{2a} = \frac{-40}{2(-0.2)} = \frac{-40}{-0.4} = \boxed{100}$$

(c) Maximum profit:

$$P(100) = 40(100) - 0.2(100)^2 - 100 = 4000 - 2000 - 100 = \boxed{1900}$$

Answer:

Max profit occurs at $x = 100$ items and the maximum profit is \$1900.

Question 2

A company manufactures and sells x units of a product per week. The revenue (in thousands of dollars) and cost (in thousands of dollars) functions are given by:

$$R(x) = 50x - 0.5x^2, \quad C(x) = 20x + 100$$

- (a) Find the profit function $P(x)$.
- (b) Determine the number of units x that maximizes the profit.
- (c) Compute the maximum profit.

Question 2 - Solution**(a) Profit Function:**

$$P(x) = R(x) - C(x) = (50x - 0.5x^2) - (20x + 100) = -0.5x^2 + 30x - 100$$

(b) Maximize Profit:

$$P'(x) = -x + 30$$

Set $P'(x) = 0$:

$$-x + 30 = 0 \quad \Rightarrow \quad x = 30$$

Second derivative:

$$P''(x) = -1 < 0$$

Hence, profit is maximized at $x = 30$.**(c) Maximum Profit:**

$$P(30) = -0.5(30)^2 + 30(30) - 100 = -450 + 900 - 100 = 350$$

Maximum profit occurs at $x = 30$ units, $P_{\max} = 350$ thousand dollars

Question 3

A company's profit $P(x)$ (in thousands of dollars) depends on the number of units x produced per week:

$$P(x) = -2x^2 + 40x - 120$$

- (a) Determine the production level x that maximizes profit.
- (b) Compute the maximum profit.
- (c) Find the production level x at which the profit is zero.

Question 3 - Solution**(a) Maximize Profit:**

$$P'(x) = \frac{d}{dx}(-2x^2 + 40x - 120) = -4x + 40$$

Set $P'(x) = 0$:

$$-4x + 40 = 0 \quad \Rightarrow \quad x = 10$$

Second derivative:

$$P''(x) = -4 < 0$$

Hence, profit is maximized at $x = 10$.**(b) Maximum Profit:**

$$P(10) = -2(10)^2 + 40(10) - 120 = -200 + 400 - 120 = 80$$

(c) Break-even Points (Profit = 0):

$$-2x^2 + 40x - 120 = 0$$

Divide through by 2:

$$x^2 - 20x + 60 = 0$$

Solve using the quadratic formula:

$$x = \frac{20 \pm \sqrt{400 - 240}}{2} = \frac{20 \pm \sqrt{160}}{2} = 10 \pm 2\sqrt{10}$$

Maximum profit occurs at $x = 10$ units, $P_{\max} = 80$ thousand dollars,
Profit is zero at $x = 10 - 2\sqrt{10}$, $10 + 2\sqrt{10}$.

Question 4

The demand for a product is modeled by

$$p(x) = 100 - 2x,$$

where $p(x)$ is the price per unit in dollars and x is the number of units sold in hundreds. The total cost of producing $100x$ units is

$$C(x) = 20x + 200 \quad \text{dollars.}$$

Require $x \geq 0$ and $p(x) \geq 0$.

- (a) Find the revenue and profit functions.
- (b) Find the production level and maximum profit in the continuous model.
- (c) Find the best whole-number quantity of units and its profit.

Question 4 - Solution**(a) Revenue and profit**

The number sold is $100x$, so revenue is $100x p(x)$:

$$R(x) = 10000x - 200x^2, \quad P(x) = -200x^2 + 9980x - 200.$$

$$P'(x) = -400x + 9980, \quad P''(x) = -400 < 0.$$

(b) Continuous maximum

The continuous maximum occurs at

$$x = \frac{9980}{400} \approx 24.950000, \quad P_{\max} \approx \$124,300.50.$$

(c) Whole units

This represents 2495.000000 units. Comparing the adjacent whole-unit counts gives 2495 units, with profit \$124,300.50. The price is nonnegative at these levels.

Question 5

A company produces and sells x units of a product per week. The weekly revenue and cost functions (in thousands of dollars) are:

$$R(x) = 200x - 2x^2, \quad C(x) = 50x + 300$$

- (a) Find the profit function $P(x)$.
- (b) Determine the number of units that maximizes the profit.
- (c) Compute the maximum profit.

Question 5 - Solution

$$P(x) = R(x) - C(x) = -2x^2 + 150x - 300.$$

In the continuous model, $P'(x) = -4x + 150 = 0$ gives $x = 37.5$ and $P''(x) = -4 < 0$.

The continuous maximum is 2512.5 thousand dollars.

For whole units, compare $x = 37$ and $x = 38$, the integers nearest the vertex:

$$P(37) = P(38) = 2512.$$

Thus the feasible maximum is

37 or 38 units,	2512 thousand dollars.
-----------------	------------------------

Question 6

The demand for a product is modeled by

$$p(x) = 150 - 3x,$$

where $p(x)$ is the price per unit in dollars and x is the number of units sold in hundreds. The total cost of producing $100x$ units is

$$C(x) = 50x + 300 \quad \text{dollars.}$$

Require $x \geq 0$ and $p(x) \geq 0$.

- (a) Find the revenue and profit functions.
- (b) Find the production level and maximum profit in the continuous model.
- (c) Find the best whole-number quantity of units and its profit.

Question 6 - Solution**(a) Revenue and profit**

The number sold is $100x$, so revenue is $100x p(x)$:

$$R(x) = 15000x - 300x^2, \quad P(x) = -300x^2 + 14950x - 300.$$

$$P'(x) = -600x + 14950, \quad P''(x) = -600 < 0.$$

(b) Continuous maximum

The continuous maximum occurs at

$$x = \frac{14950}{600} \approx 24.916667, \quad P_{\max} \approx \$185,952.08.$$

(c) Whole units

This represents 2491.666667 units. Comparing the adjacent whole-unit counts gives 2492 units, with profit \$185,952.08. The price is nonnegative at these levels.

Question 7

A company's profit $P(x)$ (in dollars) depends on the number of items x produced per week:

$$P(x) = -5x^2 + 300x - 2000$$

- (a) Determine the production level x that maximizes profit.
- (b) Compute the maximum profit.
- (c) Find the production levels x at which the profit is zero.

Question 7 - Solution**(a) Maximize Profit:**

$$P'(x) = \frac{d}{dx}(-5x^2 + 300x - 2000) = -10x + 300$$

Set $P'(x) = 0$:

$$-10x + 300 = 0 \quad \Rightarrow \quad x = 30$$

Second derivative:

$$P''(x) = -10 < 0$$

Hence, profit is maximized at $x = 30$.**(b) Maximum Profit:**

$$P(30) = -5(30)^2 + 300(30) - 2000 = -4500 + 9000 - 2000 = 2500$$

(c) Break-even Points (Profit = 0):

$$-5x^2 + 300x - 2000 = 0$$

Divide through by 5:

$$x^2 - 60x + 400 = 0$$

Solve using the quadratic formula:

$$x = \frac{60 \pm \sqrt{3600 - 1600}}{2} = \frac{60 \pm \sqrt{2000}}{2} = 30 \pm 10\sqrt{5}$$

Maximum profit occurs at $x = 30$ units, $P_{\max} = 2500$,
Profit is zero at $x = 30 - 10\sqrt{5}$, $30 + 10\sqrt{5}$.

Question 8

The weekly revenue (in thousands of dollars) for producing x units is:

$$R(x) = 120x - x^2$$

and the weekly cost is:

$$C(x) = 40x + 200$$

- (a) Determine the profit function $P(x)$.
- (b) Find the number of units x that maximizes profit.
- (c) Compute the maximum profit.

Question 8 - Solution**(a) Profit Function:**

$$P(x) = R(x) - C(x) = (120x - x^2) - (40x + 200) = -x^2 + 80x - 200$$

(b) Maximize Profit:

$$P'(x) = -2x + 80$$

Set $P'(x) = 0$:

$$-2x + 80 = 0 \quad \Rightarrow \quad x = 40$$

Second derivative:

$$P''(x) = -2 < 0$$

Hence, profit is maximized when $x = 40$.**(c) Maximum Profit:**

$$P(40) = -(40)^2 + 80(40) - 200 = -1600 + 3200 - 200 = 1400$$

Maximum profit occurs at $x = 40$ units, $P_{\max} = 1400$ thousand dollars

Question 9

The demand for a product is modeled by

$$p(x) = 80 - x,$$

where $p(x)$ is the price per unit in dollars and x is the number of units sold in hundreds. The total cost of producing $100x$ units is

$$C(x) = 20x + 100 \quad \text{dollars.}$$

Require $x \geq 0$ and $p(x) \geq 0$.

- (a) Find the revenue and profit functions.
- (b) Find the production level and maximum profit in the continuous model.
- (c) Find the best whole-number quantity of units and its profit.

Question 9 - Solution**(a) Revenue and profit**

The number sold is $100x$, so revenue is $100x p(x)$:

$$R(x) = 8000x - 100x^2, \quad P(x) = -100x^2 + 7980x - 100.$$

$$P'(x) = -200x + 7980, \quad P''(x) = -200 < 0.$$

(b) Continuous maximum

The continuous maximum occurs at

$$x = \frac{7980}{200} \approx 39.900000, \quad P_{\max} \approx \$159,101.00.$$

(c) Whole units

This represents 3990.000000 units. Comparing the adjacent whole-unit counts gives 3990 units, with profit \$159,101.00. The price is nonnegative at these levels.

Question 10

A company manufactures and sells a product. The weekly revenue and cost functions (in thousands of dollars) are:

$$R(x) = 500x - 5x^2, \quad C(x) = 200x + 500$$

- (a) Determine the profit function $P(x)$.
- (b) Find the production level x that maximizes profit.
- (c) Compute the maximum profit.

Question 10 - Solution**(a) Profit Function:**

$$P(x) = R(x) - C(x) = (500x - 5x^2) - (200x + 500) = -5x^2 + 300x - 500$$

(b) Maximize Profit:

$$P'(x) = -10x + 300$$

Set $P'(x) = 0$:

$$-10x + 300 = 0 \quad \Rightarrow \quad x = 30$$

Check the second derivative:

$$P''(x) = -10 < 0$$

Thus, profit is maximized when $x = 30$.**(c) Maximum Profit:**

$$P(30) = -5(30)^2 + 300(30) - 500 = -4500 + 9000 - 500 = 4000$$

Maximum profit occurs at $x = 30$ units, $P_{\max} = 4000$ thousand dollars