

Question 1

Use Newton's Method to approximate a root of the function

$$f(x) = x^3 - x - 1$$

starting with $x_0 = 1.5$, and compute the first two iterations: x_1 and x_2 .

Question 1 - Solution

Newton's iteration is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f(x) = x^3 - x - 1, \quad f'(x) = 3x^2 - 1.$$

Keeping full precision internally gives

$$x_1 = 1.5000000000 - \frac{0.8750000000}{5.7500000000} \approx \boxed{1.3478}$$

$$x_2 = 1.3478260870 - \frac{0.1006821731}{4.4499054820} \approx \boxed{1.3252}$$

Question 2

Use Newton's Method to approximate the root of the equation

$$x^3 - 2x - 5 = 0$$

starting with the initial guess $x_1 = 2$. Perform three iterations to compute x_2 , x_3 , and x_4 . Round your answers to four decimal places.

Question 2 - Solution

Newton's iteration is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f(x) = x^3 - 2x - 5, \quad f'(x) = 3x^2 - 2.$$

Keeping full precision internally gives

$$x_2 = 2.0000000000 - \frac{-1.0000000000}{10.0000000000} \approx \boxed{2.1000}$$

$$x_3 = 2.1000000000 - \frac{0.0610000000}{11.2300000000} \approx \boxed{2.0946}$$

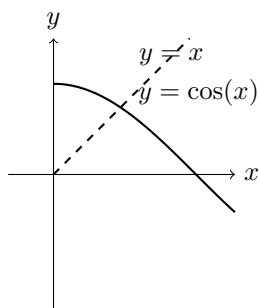
$$x_4 = 2.0945681211 - \frac{0.0001857232}{11.1616468418} \approx \boxed{2.0946}$$

Question 3

Use Newton's Method to approximate a solution to the equation:

$$\cos(x) = x$$

Start with an initial guess of $x_1 = 0.5$, and compute the next three iterations: x_2 , x_3 , and x_4 , rounding answers to four decimal places.



Question 3 - Solution

Newton's iteration is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f(x) = \cos x - x, \quad f'(x) = -\sin x - 1.$$

Keeping full precision internally gives

$$x_2 = 0.5000000000 - \frac{0.3775825619}{-1.4794255386} \approx \boxed{0.7552}$$

$$x_3 = 0.7552224171 - \frac{-0.0271033119}{-1.6854506318} \approx \boxed{0.7391}$$

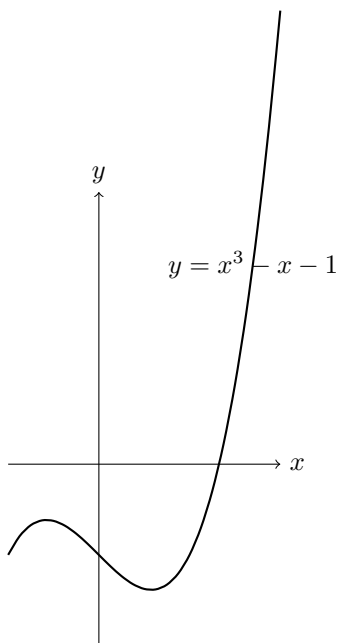
$$x_4 = 0.7391416661 - \frac{-0.0000946154}{-1.6736538108} \approx \boxed{0.7391}$$

Question 4

Use Newton's Method to approximate a root of the equation:

$$x^3 - x - 1 = 0$$

Start with an initial guess of $x_1 = 1$, and compute the next three iterations: x_2 , x_3 , and x_4 , rounding answers to four decimal places.



Question 4 - Solution

Newton's iteration is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f(x) = x^3 - x - 1, \quad f'(x) = 3x^2 - 1.$$

Keeping full precision internally gives

$$x_2 = 1.0000000000 - \frac{-1.0000000000}{2.0000000000} \approx \boxed{1.5000}$$

$$x_3 = 1.5000000000 - \frac{0.8750000000}{5.7500000000} \approx \boxed{1.3478}$$

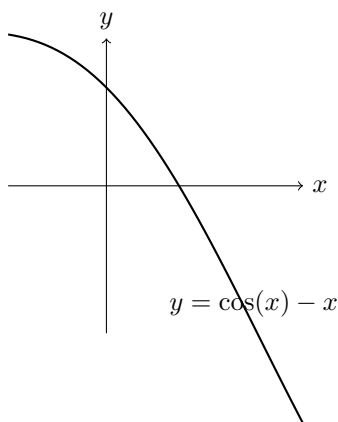
$$x_4 = 1.3478260870 - \frac{0.1006821731}{4.4499054820} \approx \boxed{1.3252}$$

Question 5

Use Newton's Method to approximate a root of the equation:

$$\cos(x) - x = 0$$

Start with an initial guess of $x_1 = 0.5$, and compute the next three iterations: x_2 , x_3 , and x_4 , rounding to four decimal places.



Question 5 - Solution

Newton's iteration is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f(x) = \cos x - x, \quad f'(x) = -\sin x - 1.$$

Keeping full precision internally gives

$$x_2 = 0.5000000000 - \frac{0.3775825619}{-1.4794255386} \approx \boxed{0.7552}$$

$$x_3 = 0.7552224171 - \frac{-0.0271033119}{-1.6854506318} \approx \boxed{0.7391}$$

$$x_4 = 0.7391416661 - \frac{-0.0000946154}{-1.6736538108} \approx \boxed{0.7391}$$

Question 6

Use Newton's Method to approximate a root of the equation:

$$x^3 - 2x - 5 = 0$$

Start with an initial guess of $x_1 = 2$, and compute the next three approximations: x_2 , x_3 , and x_4 . Round answers to four decimal places.

Question 6 - Solution

Newton's iteration is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f(x) = x^3 - 2x - 5, \quad f'(x) = 3x^2 - 2.$$

Keeping full precision internally gives

$$x_2 = 2.0000000000 - \frac{-1.0000000000}{10.0000000000} \approx \boxed{2.1000}$$

$$x_3 = 2.1000000000 - \frac{0.0610000000}{11.2300000000} \approx \boxed{2.0946}$$

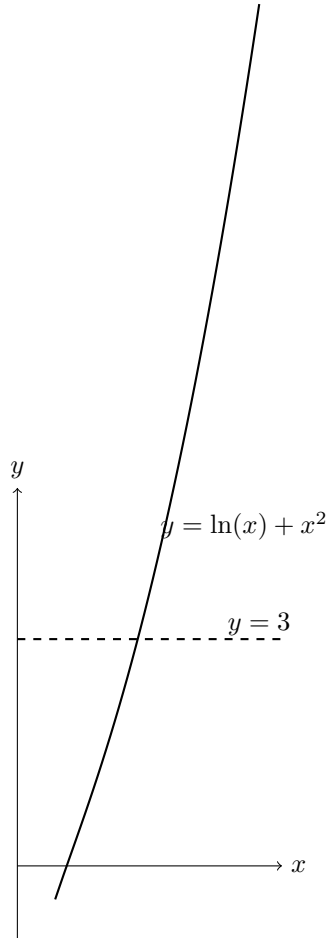
$$x_4 = 2.0945681211 - \frac{0.0001857232}{11.1616468418} \approx \boxed{2.0946}$$

Question 7

Use Newton's Method to approximate a solution to the equation:

$$\ln(x) + x^2 = 3$$

Start with an initial guess of $x_1 = 1.5$, and compute x_2 , x_3 , and x_4 . Round each to four decimal places.



Question 7 - Solution

Newton's iteration is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f(x) = \ln x + x^2 - 3, \quad f'(x) = 1/x + 2x.$$

Keeping full precision internally gives

$$x_2 = 1.5000000000 - \frac{-0.3445348919}{3.6666666667} \approx \boxed{1.5940}$$

$$x_3 = 1.5939640614 - \frac{0.0069454631}{3.8152948397} \approx \boxed{1.5921}$$

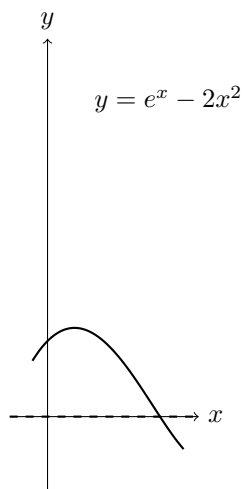
$$x_4 = 1.5921436351 - \frac{0.0000026613}{3.8123713061} \approx \boxed{1.5921}$$

Question 8

Approximate a root of the equation:

$$e^x - 2x^2 = 0$$

using Newton's Method. Start with $x_1 = 0.5$, and compute x_2 , x_3 , and x_4 to four decimal places.



Question 8 - Solution

Newton's iteration is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f(x) = e^x - 2x^2, \quad f'(x) = e^x - 4x.$$

Keeping full precision internally gives

$$x_2 = 0.5000000000 - \frac{1.1487212707}{-0.3512787293} \approx \boxed{3.7701}$$

$$x_3 = 3.7701133740 - \frac{14.9574735817}{28.3045297918} \approx \boxed{3.2417}$$

$$x_4 = 3.2416653238 - \frac{4.5594909448}{12.6096177926} \approx \boxed{2.8801}$$

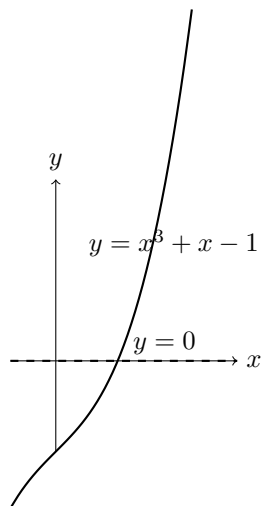
These are the requested iterates; three steps do not yet give an accurate root. Continue iterating if a converged root is required.

Question 9

Approximate a solution to the equation:

$$x^3 + x - 1 = 0$$

using Newton's Method. Start with $x_1 = 0.5$, and compute x_2 , x_3 , and x_4 , rounding each to four decimal places.



Question 9 - Solution

Newton's iteration is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f(x) = x^3 + x - 1, \quad f'(x) = 3x^2 + 1.$$

Keeping full precision internally gives

$$x_2 = 0.5000000000 - \frac{-0.3750000000}{1.7500000000} \approx \boxed{0.7143}$$

$$x_3 = 0.7142857143 - \frac{0.0787172012}{2.5306122449} \approx \boxed{0.6832}$$

$$x_4 = 0.6831797235 - \frac{0.0020432938}{2.4002036038} \approx \boxed{0.6823}$$

Question 10

Consider the equation

$$x^3 - 3x + 1 = 0.$$

- (a) Explain why Newton's method fails with the initial guess $x_1 = 1$.
- (b) Restart with $x_1 = 0$, and compute x_2 , x_3 , and x_4 . Round the answers to four decimal places.

Question 10 - Solution

At $x_1 = 1$, $f(1) = -1$ and $f'(1) = 0$, so the Newton step is undefined. Restart at $x_1 = 0$. Newton's iteration is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f(x) = x^3 - 3x + 1, \quad f'(x) = 3x^2 - 3.$$

Keeping full precision internally gives

$$x_2 = 0.0000000000 - \frac{1.0000000000}{-3.0000000000} \approx \boxed{0.3333}$$

$$x_3 = 0.3333333333 - \frac{0.0370370370}{-2.6666666667} \approx \boxed{0.3472}$$

$$x_4 = 0.3472222222 - \frac{0.0001955804}{-2.6383101852} \approx \boxed{0.3473}$$