

Question 1

Let $f(x) = x^3$.

- (a) Use differentials to approximate the change in f as x changes from 2 to 2.05.
- (b) Compare your result to the actual change $f(2.05) - f(2)$.

Question 1 - Solution

We are given:

$$f(x) = x^3, \quad f'(x) = 3x^2$$

(a) Use:

$$df = f'(x) dx$$

Let $x = 2$, $dx = 0.05$:

$$f'(2) = 3(2)^2 = 12$$

$$df = 12 \cdot 0.05 = \boxed{0.6}$$

So, the approximate change in f is $\boxed{0.6}$

(b) Actual change:

$$f(2.05) = (2.05)^3 = 8.615125, \quad f(2) = 8$$

$$\text{Actual change} = 8.615125 - 8 = \boxed{0.615125}$$

Conclusion: The differential estimate 0.6 is close to the actual change 0.615125

Question 2

Suppose the radius r of a sphere is measured to be 5 cm with a possible error of 0.1 cm.

- (a) Use differentials to approximate the possible error in the calculated volume of the sphere.
- (b) Estimate the relative error and percentage error in the volume.

Question 2 - Solution

We use the volume formula of a sphere:

$$V = \frac{4}{3}\pi r^3$$

(a) **Approximate error using differentials:**

$$dV = \frac{dV}{dr} dr = 4\pi r^2 dr$$

$$dV = 4\pi(5)^2(0.1) = 4\pi(25)(0.1) = 10\pi \approx 31.42 \text{ cm}^3$$

(b) **Relative error:**

$$\frac{dV}{V} = \frac{10\pi}{\frac{4}{3}\pi(5)^3} = \frac{10\pi}{\frac{500}{3}\pi} = \frac{10 \cdot 3}{500} = \frac{30}{500} = 0.06$$

Percentage error:

$$0.06 \times 100 = \boxed{6\%}$$

Final Answer:

- Approximate error in volume: $\boxed{31.42 \text{ cm}^3}$
- Relative error: $\boxed{0.06}$
- Percentage error: $\boxed{6\%}$

Question 3

The length of the side of a cube is measured to be 10 cm with a possible error of 0.2 cm.

- (a) Use differentials to approximate the maximum error in the calculated surface area of the cube.
- (b) Estimate the relative error and the percentage error in the surface area.

Question 3 - Solution

Let the surface area of the cube be:

$$S = 6x^2$$

where x is the length of a side.

(a) **Approximate error using differentials:**

$$dS = \frac{dS}{dx} dx = 12x dx$$

$$dS = 12(10)(0.2) = 24 \text{ cm}^2$$

(b) **Relative error:**

$$\frac{dS}{S} = \frac{24}{6(10)^2} = \frac{24}{600} = 0.04$$

Percentage error:

$$0.04 \times 100 = \boxed{4\%}$$

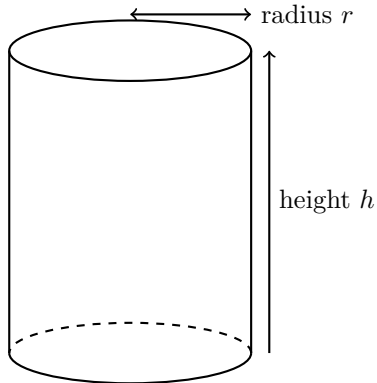
Final Answer:

- Maximum error in surface area: $\boxed{24 \text{ cm}^2}$
- Relative error: $\boxed{0.04}$
- Percentage error: $\boxed{4\%}$

Question 4

A right circular cylinder has a height of 12 cm, and its radius is measured to be 5 cm with a possible error of 0.1 cm.

- (a) Use differentials to approximate the maximum error in the calculated volume of the cylinder.
- (b) Estimate the relative error and percentage error in the volume.



Question 4 - Solution

The volume of a right circular cylinder is given by:

$$V = \pi r^2 h$$

Given: $r = 5$ cm, $dr = 0.1$ cm, $h = 12$ cm

(a) Use differentials to approximate the error:

$$dV = \frac{dV}{dr} \cdot dr = 2\pi r h \cdot dr$$

$$dV = 2\pi(5)(12)(0.1) = 12\pi \text{ cm}^3 \approx \boxed{37.70 \text{ cm}^3}$$

(b) Estimate the relative error and percentage error:

$$V = \pi r^2 h = \pi(5)^2(12) = 300\pi \approx 942.48 \text{ cm}^3$$

$$\text{Relative error} = \frac{dV}{V} = \frac{12\pi}{300\pi} = \frac{12}{300} = 0.04$$

$$\text{Percentage error} = 0.04 \times 100 = \boxed{4\%}$$

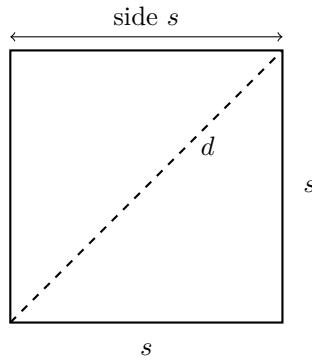
Final Answer:

- Maximum error in volume: $\boxed{37.70 \text{ cm}^3}$
- Relative error: $\boxed{0.04}$
- Percentage error: $\boxed{4\%}$

Question 5

The length of the diagonal d of a square is measured to be 10 cm with a possible error of 0.2 cm. Use differentials to approximate:

- (a) The maximum error in computing the area of the square.
- (b) The relative and percentage errors.



Question 5 - Solution

We are given that the diagonal of a square is $d = 10$ cm with an error of $dd = 0.2$ cm. The area of a square in terms of its diagonal is:

$$s = \frac{d}{\sqrt{2}}, \quad A = s^2 = \left(\frac{d}{\sqrt{2}} \right)^2 = \frac{d^2}{2}$$

Differentiate:

$$dA = \frac{d}{dx} \left(\frac{d^2}{2} \right) \cdot dd = d \cdot dd$$

(a) **Maximum error in the area:**

$$dA = d \cdot dd = 10 \cdot 0.2 = \boxed{2 \text{ cm}^2}$$

(b) **Relative and percentage error:**

Actual area:

$$A = \frac{10^2}{2} = 50 \text{ cm}^2$$

Relative error:

$$\frac{dA}{A} = \frac{2}{50} = 0.04$$

Percentage error:

$$0.04 \times 100 = \boxed{4\%}$$

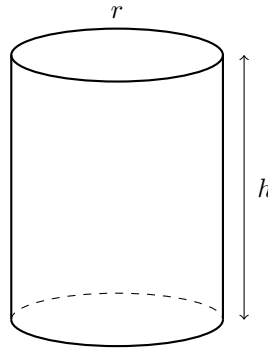
Final Answer:

- Maximum error in area: $\boxed{2 \text{ cm}^2}$
- Relative error: $\boxed{0.04}$
- Percentage error: $\boxed{4\%}$

Question 6

The radius r of a right circular cylinder is measured to be 5 cm with a possible error of 0.1 cm. The height h is 20 cm with a possible error of 0.2 cm.

- (a) Use differentials to approximate the maximum error in calculating the volume of the cylinder.
- (b) Find the relative and percentage errors in the volume.



Question 6 - Solution

The volume of a cylinder is:

$$V = \pi r^2 h$$

We compute the differential dV :

$$dV = \frac{\partial V}{\partial r} dr + \frac{\partial V}{\partial h} dh = 2\pi r h dr + \pi r^2 dh$$

Given:

$$r = 5 \text{ cm}, \quad dr = 0.1 \text{ cm}, \quad h = 20 \text{ cm}, \quad dh = 0.2 \text{ cm}$$

Substitute into the differential:

$$dV = 2\pi(5)(20)(0.1) + \pi(5)^2(0.2) = 20\pi + 5\pi = \boxed{25\pi \approx 78.54 \text{ cm}^3}$$

(a) Maximum error:

$$\boxed{dV \approx 78.54 \text{ cm}^3}$$

(b) Relative and percentage error:

Actual volume:

$$V = \pi(5)^2(20) = 500\pi \approx 1570.8 \text{ cm}^3$$

Relative error:

$$\frac{dV}{V} = \frac{25\pi}{500\pi} = \frac{1}{20} = 0.05$$

Percentage error:

$$0.05 \times 100 = \boxed{5\%}$$

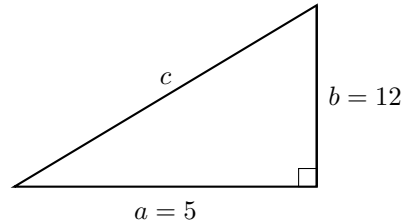
Final Answers:

- Maximum error in volume: $\boxed{25\pi \approx 78.54 \text{ cm}^3}$
- Relative error: $\boxed{0.05}$
- Percentage error: $\boxed{5\%}$

Question 7

A right triangle has one leg of fixed length 5 cm. The other leg is measured to be 12 cm with a possible measurement error of 0.1 cm.

- (a) Use differentials to approximate the maximum error in computing the hypotenuse.
- (b) Find the relative and percentage error in the hypotenuse.



Question 7 - Solution

We are given that $a = 5$ is fixed and $b = 12$ with a possible error $db = 0.1$. The hypotenuse is:

$$c = \sqrt{a^2 + b^2}$$

Differentiate both sides:

$$dc = \frac{1}{2}(a^2 + b^2)^{-1/2} \cdot 2b db = \frac{b}{\sqrt{a^2 + b^2}} \cdot db$$

Substitute known values:

$$a = 5, \quad b = 12, \quad db = 0.1$$
$$c = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

$$dc = \frac{12}{13} \cdot 0.1 \approx \boxed{0.0923}$$

(a) **Maximum error:**

$$\boxed{dc \approx 0.0923 \text{ cm}}$$

(b) **Relative and percentage error:**

$$\text{Relative error} = \frac{dc}{c} = \frac{0.0923}{13} \approx 0.0071$$

$$\text{Percentage error} = 0.0071 \times 100 \approx \boxed{0.71\%}$$

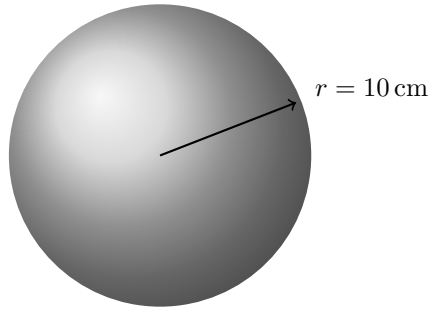
Final Answers:

- Maximum error in hypotenuse: $\boxed{0.0923 \text{ cm}}$
- Relative error: $\boxed{0.0071}$
- Percentage error: $\boxed{0.71\%}$

Question 8

The radius of a spherical balloon is measured to be 10 cm with a possible error of 0.05 cm.

- (a) Use differentials to estimate the maximum error in calculating the volume of the balloon.
- (b) Estimate the relative and percentage error in the volume.



Question 8 - Solution

We are given:

$$r = 10 \text{ cm}, \quad dr = 0.05 \text{ cm}$$

The volume of a sphere is:

$$V = \frac{4}{3}\pi r^3$$

Differentiate using differentials:

$$dV = \frac{d}{dr} \left(\frac{4}{3}\pi r^3 \right) \cdot dr = 4\pi r^2 \cdot dr$$

Substitute known values:

$$dV = 4\pi(10)^2 \cdot 0.05 = 4\pi \cdot 100 \cdot 0.05 = 20\pi \approx \boxed{62.83 \text{ cm}^3}$$

(a) Maximum error in volume:

$$\boxed{dV \approx 62.83 \text{ cm}^3}$$

(b) Relative and percentage error:

Actual volume:

$$V = \frac{4}{3}\pi(10)^3 = \frac{4000}{3}\pi \approx 4188.79 \text{ cm}^3$$

$$\text{Relative error} = \frac{dV}{V} \approx \frac{62.83}{4188.79} \approx 0.015$$

$$\text{Percentage error} = 0.015 \times 100 = \boxed{1.5\%}$$

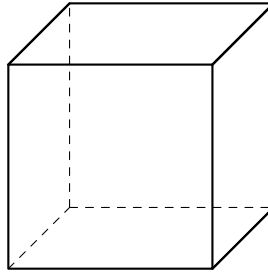
Final Answers:

- Maximum error in volume: $\boxed{62.83 \text{ cm}^3}$
- Relative error: $\boxed{0.015}$
- Percentage error: $\boxed{1.5\%}$

Question 9

The side of a cube is measured to be 5 cm with a possible error of 0.1 cm.

- (a) Use differentials to estimate the maximum error in computing the volume of the cube.
- (b) Estimate the relative and percentage error in the volume.



$$s = 5 \text{ cm}$$

Question 9 - Solution

We are given:

$$s = 5 \text{ cm}, \quad ds = 0.1 \text{ cm}$$

The volume of a cube is:

$$V = s^3$$

Differentiate using differentials:

$$dV = \frac{d}{ds}(s^3) ds = 3s^2 ds$$

Substitute values:

$$dV = 3(5)^2(0.1) = 3 \cdot 25 \cdot 0.1 = \boxed{7.5 \text{ cm}^3}$$

(a) **Maximum error in volume:**

$$\boxed{7.5 \text{ cm}^3}$$

(b) **Relative and percentage error:**

$$V = 5^3 = 125 \text{ cm}^3$$

$$\text{Relative error} = \frac{dV}{V} = \frac{7.5}{125} = 0.06$$

$$\text{Percentage error} = 0.06 \times 100 = \boxed{6\%}$$

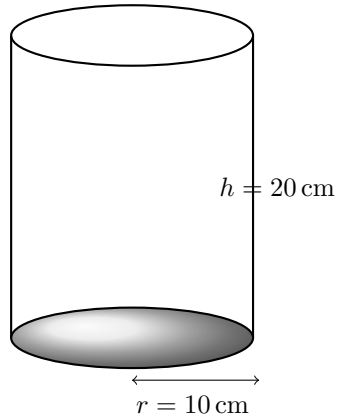
Final Answers:

- Maximum error in volume: $\boxed{7.5 \text{ cm}^3}$
- Relative error: $\boxed{0.06}$
- Percentage error: $\boxed{6\%}$

Question 10

The radius of a right circular cylinder is measured as $r = 10$ cm with a possible error of 0.2 cm, and the height is measured as $h = 20$ cm with a possible error of 0.3 cm.

- (a) Use differentials to estimate the maximum error in the volume of the cylinder.
- (b) Estimate the percentage error in the volume.



Question 10 - Solution

We are given:

$$r = 10 \text{ cm}, \quad dr = 0.2 \text{ cm}, \quad h = 20 \text{ cm}, \quad dh = 0.3 \text{ cm}$$

The volume of a cylinder is:

$$V = \pi r^2 h$$

Use differentials:

$$dV = \frac{\partial}{\partial r} dr + \frac{\partial}{\partial h} dh = (2\pi r h) \cdot dr + (\pi r^2) \cdot dh$$

Substitute values:

$$\begin{aligned} dV &= 2\pi(10)(20)(0.2) + \pi(10)^2(0.3) \\ &= 2\pi(200)(0.2) + \pi(100)(0.3) = \pi(80) + \pi(30) = \pi(110) \end{aligned}$$

$$\boxed{dV \approx 345.58 \text{ cm}^3}$$

(a) **Estimated maximum error:** $\boxed{345.58 \text{ cm}^3}$

(b) **Percentage error:**

$$V = \pi r^2 h = \pi(100)(20) = 2000\pi \approx 6283.19 \text{ cm}^3$$

$$\text{Percentage error} = \frac{345.58}{6283.19} \times 100 \approx \boxed{5.5\%}$$

Final Answers:

- Maximum error in volume: $\boxed{345.58 \text{ cm}^3}$
- Percentage error: $\boxed{5.5\%}$