

Question 1

Evaluate the limit:

$$\lim_{x \rightarrow \infty} \frac{6x^3 - 4x^2 + 10}{3x^3 + 5x - 7}$$

Question 1 - Solution

We are given:

$$\lim_{x \rightarrow \infty} \frac{6x^3 - 4x^2 + 10}{3x^3 + 5x - 7}$$

Step 1: Identify the dominant terms.

As $x \rightarrow \infty$, the highest degree terms dominate: Numerator: $6x^3$, Denominator: $3x^3$

So we focus on:

$$\frac{6x^3}{3x^3} = 2$$

Step 2: Divide all terms by x^3 :

$$\lim_{x \rightarrow \infty} \frac{6 - \frac{4}{x} + \frac{10}{x^3}}{3 + \frac{5}{x^2} - \frac{7}{x^3}} = \frac{6}{3} = 2$$

Conclusion:

$\lim_{x \rightarrow \infty} \frac{6x^3 - 4x^2 + 10}{3x^3 + 5x - 7} = 2$
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Question 2

Evaluate the limit:

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x} - x)$$

Question 2 - Solution

We are given:

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 5x} - x)$$

This expression is of the indeterminate form $\infty - \infty$, so we need to rationalize:

Multiply by the conjugate:

$$(\sqrt{x^2 + 5x} - x) \cdot \frac{\sqrt{x^2 + 5x} + x}{\sqrt{x^2 + 5x} + x}$$

This gives:

$$\frac{(\sqrt{x^2 + 5x})^2 - x^2}{\sqrt{x^2 + 5x} + x} = \frac{x^2 + 5x - x^2}{\sqrt{x^2 + 5x} + x} = \frac{5x}{\sqrt{x^2 + 5x} + x}$$

Now divide numerator and denominator by x :

$$\frac{5}{\sqrt{1 + \frac{5}{x}} + 1}$$

As $x \rightarrow \infty$, $\frac{5}{x} \rightarrow 0$, so:

$$\frac{5}{\sqrt{1+0}+1} = \frac{5}{1+1} = \frac{5}{2}$$

Final Answer:

$$\boxed{\frac{5}{2}}$$

Question 3

Evaluate the limit:

$$\lim_{x \rightarrow \infty} \frac{3x^2 - x + 4}{5x^2 + 7x - 9}$$

Question 3 - Solution

We are given:

$$\lim_{x \rightarrow \infty} \frac{3x^2 - x + 4}{5x^2 + 7x - 9}$$

To evaluate this limit, we divide both numerator and denominator by x^2 , the highest power of x :

$$\lim_{x \rightarrow \infty} \frac{3 - \frac{1}{x} + \frac{4}{x^2}}{5 + \frac{7}{x} - \frac{9}{x^2}}$$

As $x \rightarrow \infty$, the terms $\frac{1}{x}, \frac{4}{x^2}, \frac{7}{x}, \frac{9}{x^2} \rightarrow 0$, so we get:

$$\frac{3 - 0 + 0}{5 + 0 - 0} = \frac{3}{5}$$

Final Answer:

$$\boxed{\frac{3}{5}}$$

Question 4

Evaluate the limit:

$$\lim_{x \rightarrow -\infty} \frac{6x^3 + 4x - 1}{-2x^3 + 7x^2 + 5}$$

Question 4 - Solution

We are given:

$$\lim_{x \rightarrow -\infty} \frac{6x^3 + 4x - 1}{-2x^3 + 7x^2 + 5}$$

We divide both the numerator and denominator by x^3 , the highest power of x :

$$= \lim_{x \rightarrow -\infty} \frac{6 + \frac{4}{x^2} - \frac{1}{x^3}}{-2 + \frac{7}{x} + \frac{5}{x^3}}$$

As $x \rightarrow -\infty$, all terms with negative exponents approach zero:

$$= \frac{6}{-2} = -3$$

Final Answer:

$$\boxed{-3}$$

Question 5

Determine the limit:

$$\lim_{x \rightarrow \infty} \frac{5x^4 - 3x^2 + x + 1}{2x^3 + 4x - 7}$$

Question 5 - Solution

We are given:

$$\lim_{x \rightarrow \infty} \frac{5x^4 - 3x^2 + x + 1}{2x^3 + 4x - 7}$$

Step 1: Compare the degrees of the numerator and denominator.

Degree of numerator: 4 , Degree of denominator: 3

Since the degree of the numerator is greater than the degree of the denominator, the limit is infinite (the function diverges).

Step 2: Determine the sign of the infinity.

As $x \rightarrow \infty$, both numerator and denominator are positive:

$$\frac{5x^4 + \text{smaller terms}}{2x^3 + \text{smaller terms}} > 0$$

So the limit tends to:

$$\boxed{+\infty}$$

Question 6

Evaluate the limit:

$$\lim_{x \rightarrow \infty} \frac{3x^5 - 2x^2 + 1}{x^5 + 4x^3 - x}$$

Question 6 - Solution

We are given:

$$\lim_{x \rightarrow \infty} \frac{3x^5 - 2x^2 + 1}{x^5 + 4x^3 - x}$$

Step 1: Divide numerator and denominator by the highest power of x , which is x^5 :

$$= \lim_{x \rightarrow \infty} \frac{3 - \frac{2}{x^3} + \frac{1}{x^5}}{1 + \frac{4}{x^2} - \frac{1}{x^4}}$$

Step 2: As $x \rightarrow \infty$, all terms with negative powers of x go to zero:

$$= \frac{3 - 0 + 0}{1 + 0 - 0} = \boxed{3}$$

Question 7

Evaluate the limit:

$$\lim_{x \rightarrow -\infty} \frac{5x^3 + x^2 - 7x + 1}{-2x^3 + 3x - 9}$$

Question 7 - Solution

We are given:

$$\lim_{x \rightarrow -\infty} \frac{5x^3 + x^2 - 7x + 1}{-2x^3 + 3x - 9}$$

Step 1: Factor out the highest degree of x from both numerator and denominator:

$$= \lim_{x \rightarrow -\infty} \frac{x^3 \left(5 + \frac{1}{x} - \frac{7}{x^2} + \frac{1}{x^3} \right)}{x^3 \left(-2 + \frac{3}{x^2} - \frac{9}{x^3} \right)}$$

Cancel the common factor of x^3 :

$$= \lim_{x \rightarrow -\infty} \frac{5 + \frac{1}{x} - \frac{7}{x^2} + \frac{1}{x^3}}{-2 + \frac{3}{x^2} - \frac{9}{x^3}}$$

Step 2: Let $x \rightarrow -\infty$. All terms with negative powers vanish:

$$= \frac{5 + 0 - 0 + 0}{-2 + 0 - 0} = \boxed{-\frac{5}{2}}$$

Question 8

Determine the limit:

$$\lim_{x \rightarrow \infty} \frac{6x^2 - x + 8}{\sqrt{x^4 + 3x^2 + 1}}$$

Question 8 - Solution

We are given:

$$\lim_{x \rightarrow \infty} \frac{6x^2 - x + 8}{\sqrt{x^4 + 3x^2 + 1}}$$

Step 1: Rewrite the denominator using properties of radicals:

$$\sqrt{x^4 + 3x^2 + 1} = x^2 \sqrt{1 + \frac{3}{x^2} + \frac{1}{x^4}}$$

So the entire expression becomes:

$$\frac{6x^2 - x + 8}{x^2 \sqrt{1 + \frac{3}{x^2} + \frac{1}{x^4}}} = \frac{x^2 (6 - \frac{1}{x} + \frac{8}{x^2})}{x^2 \sqrt{1 + \frac{3}{x^2} + \frac{1}{x^4}}}$$

Cancel x^2 :

$$= \frac{6 - \frac{1}{x} + \frac{8}{x^2}}{\sqrt{1 + \frac{3}{x^2} + \frac{1}{x^4}}}$$

Step 2: Let $x \rightarrow \infty$:

$$= \frac{6 - 0 + 0}{\sqrt{1 + 0 + 0}} = \frac{6}{1} = \boxed{6}$$

Question 9

Evaluate the limit:

$$\lim_{x \rightarrow -\infty} \frac{2x^3 + x^2 - 5}{x^3 - 4x + 1}$$

Question 9 - Solution

We are given:

$$\lim_{x \rightarrow -\infty} \frac{2x^3 + x^2 - 5}{x^3 - 4x + 1}$$

Step 1: Factor out the highest power of x (which is x^3) from both numerator and denominator:

$$= \lim_{x \rightarrow -\infty} \frac{x^3 \left(2 + \frac{1}{x} - \frac{5}{x^3}\right)}{x^3 \left(1 - \frac{4}{x^2} + \frac{1}{x^3}\right)}$$

Cancel the common x^3 terms:

$$= \lim_{x \rightarrow -\infty} \frac{2 + \frac{1}{x} - \frac{5}{x^3}}{1 - \frac{4}{x^2} + \frac{1}{x^3}}$$

Step 2: Take the limit as $x \rightarrow -\infty$. All terms with $\frac{1}{x^n} \rightarrow 0$:

$$= \frac{2 + 0 - 0}{1 - 0 + 0} = \boxed{2}$$

Question 10

Evaluate the limit:

$$\lim_{x \rightarrow \infty} \frac{5x^2 + 3x - 7}{2x^2 - x + 4}$$

Question 10 - Solution

We are given:

$$\lim_{x \rightarrow \infty} \frac{5x^2 + 3x - 7}{2x^2 - x + 4}$$

Step 1: Factor out the highest power of x (which is x^2) from both the numerator and denominator:

$$= \lim_{x \rightarrow \infty} \frac{x^2 \left(5 + \frac{3}{x} - \frac{7}{x^2}\right)}{x^2 \left(2 - \frac{1}{x} + \frac{4}{x^2}\right)}$$

Step 2: Cancel the common factor x^2 :

$$= \lim_{x \rightarrow \infty} \frac{5 + \frac{3}{x} - \frac{7}{x^2}}{2 - \frac{1}{x} + \frac{4}{x^2}}$$

Step 3: Take the limit as $x \rightarrow \infty$. Terms with $\frac{1}{x^n} \rightarrow 0$:

$$= \frac{5 + 0 - 0}{2 - 0 + 0} = \boxed{\frac{5}{2}}$$