

Question 1

Evaluate the following limit and describe the behavior of the function near the point of discontinuity:

$$\lim_{x \rightarrow 1^-} \frac{3}{(x-1)^2}$$

Question 1 - Solution

We are asked to evaluate:

$$\lim_{x \rightarrow 1^-} \frac{3}{(x-1)^2}$$

Step 1: Observe the behavior of the denominator

As $x \rightarrow 1^-$, the value of $x - 1$ becomes a small negative number. Squaring it makes it a small positive number:

$$(x-1)^2 > 0 \quad \text{but very small}$$

So the expression becomes:

$$\frac{3}{\text{small positive number}} \rightarrow +\infty$$

Conclusion:

The function grows without bound as x approaches 1 from the left.

$$\boxed{\lim_{x \rightarrow 1^-} \frac{3}{(x-1)^2} = \infty}$$

Question 2

Evaluate the limit:

$$\lim_{x \rightarrow 0^+} \frac{\tan x}{x}.$$

Question 2 - Solution

We evaluate

$$\lim_{x \rightarrow 0^+} \frac{\tan x}{x}.$$

Method 1: Local Approximation

As $x \rightarrow 0$, the Taylor expansion of $\tan x$ is

$$\tan x = x + \frac{x^3}{3} + \cdots.$$

Thus,

$$\frac{\tan x}{x} = \frac{x + \frac{x^3}{3} + \cdots}{x} = 1 + \frac{x^2}{3} + \cdots.$$

As $x \rightarrow 0^+$, the higher-order terms approach 0, so

$$\lim_{x \rightarrow 0^+} \frac{\tan x}{x} = 1.$$

Method 2: Rewrite Using Sine and Cosine

Recall that

$$\tan x = \frac{\sin x}{\cos x}.$$

Then,

$$\frac{\tan x}{x} = \frac{\sin x}{x} \cdot \frac{1}{\cos x}.$$

Now evaluate each factor as $x \rightarrow 0^+$:

$$\lim_{x \rightarrow 0^+} \frac{\sin x}{x} = 1, \quad \lim_{x \rightarrow 0^+} \cos x = 1.$$

Therefore,

$$\lim_{x \rightarrow 0^+} \frac{\tan x}{x} = 1 \cdot \frac{1}{1} = 1.$$

Conclusion

$$\boxed{\lim_{x \rightarrow 0^+} \frac{\tan x}{x} = 1}$$

Question 3

Evaluate the following limit:

$$\lim_{x \rightarrow 3^-} \frac{1}{(x-3)^2}$$

Question 3 - Solution

We are given:

$$\lim_{x \rightarrow 3^-} \frac{1}{(x-3)^2}$$

Step 1: Analyze the behavior of the denominator

As $x \rightarrow 3^-$, we are approaching 3 from the left. The expression $x - 3$ becomes a small negative number. But since it is squared:

$$(x-3)^2 \rightarrow 0^+ \quad (\text{very small positive number})$$

Step 2: Behavior of the function

Since we are dividing by a very small positive number, the entire expression becomes very large:

$$\frac{1}{(x-3)^2} \rightarrow +\infty$$

Conclusion:

$$\boxed{\lim_{x \rightarrow 3^-} \frac{1}{(x-3)^2} = \infty}$$

Question 4

Evaluate the following limit:

$$\lim_{x \rightarrow 1^+} \frac{5}{x - 1}$$

Question 4 - Solution

We are given:

$$\lim_{x \rightarrow 1^+} \frac{5}{x - 1}$$

Step 1: Understand the behavior of the denominator

As $x \rightarrow 1^+$, we are approaching 1 from the right. This means $x - 1$ is a small positive number.

Step 2: Behavior of the whole expression

$$\frac{5}{x - 1} \rightarrow \frac{5}{\text{small positive number}} \rightarrow +\infty$$

Conclusion:

$$\boxed{\lim_{x \rightarrow 1^+} \frac{5}{x - 1} = \infty}$$

Question 5

Evaluate the following limit:

$$\lim_{x \rightarrow 0^-} \frac{1}{x^2}$$

Question 5 - Solution

We are given:

$$\lim_{x \rightarrow 0^-} \frac{1}{x^2}$$

Step 1: Analyze the expression

As $x \rightarrow 0^-$, x is approaching 0 from the left. So x is negative, but very close to 0. Now, consider the square of x :

$$x^2 > 0 \text{ for all } x \neq 0$$

Even though x is negative, x^2 is positive. As $x \rightarrow 0^-$, $x^2 \rightarrow 0^+$

Step 2: Behavior of the expression

$$\frac{1}{x^2} \rightarrow \frac{1}{\text{small positive number}} \rightarrow \infty$$

Conclusion:

$$\boxed{\lim_{x \rightarrow 0^-} \frac{1}{x^2} = \infty}$$

Question 6

Evaluate the limit:

$$\lim_{x \rightarrow -2^+} \frac{4}{(x+2)^3}$$

Question 6 - Solution

We are asked to evaluate:

$$\lim_{x \rightarrow -2^+} \frac{4}{(x+2)^3}$$

Step 1: Analyze the denominator

As $x \rightarrow -2^+$, we approach -2 from the right, so:

$$x + 2 \rightarrow 0^+ \quad (\text{a small positive number})$$

Now consider the cube:

$$(x + 2)^3 \rightarrow 0^+ \quad (\text{still positive, but very small})$$

Step 2: Behavior of the expression

Since the numerator is positive and the denominator approaches a very small positive number:

$$\frac{4}{(x+2)^3} \rightarrow +\infty$$

Conclusion:

$$\boxed{\lim_{x \rightarrow -2^+} \frac{4}{(x+2)^3} = \infty}$$

Question 7

Evaluate the limit:

$$\lim_{x \rightarrow 1^-} \frac{x^2 + 3x - 4}{x - 1}$$

Question 7 - Solution

We are given:

$$\lim_{x \rightarrow 1^-} \frac{x^2 + 3x - 4}{x - 1}$$

Step 1: Factor the numerator

Factor the quadratic:

$$x^2 + 3x - 4 = (x + 4)(x - 1)$$

So the expression becomes:

$$\frac{(x + 4)(x - 1)}{x - 1}$$

We can cancel $x - 1$ when $x \neq 1$:

$$= x + 4 \quad \text{for } x \neq 1$$

So the original expression simplifies to $x + 4$, and we can evaluate the limit directly.

Step 2: Evaluate the limit

Since we simplified the expression:

$$\lim_{x \rightarrow 1^-} (x + 4) = 1 + 4 = 5$$

Conclusion:

$$\boxed{\lim_{x \rightarrow 1^-} \frac{x^2 + 3x - 4}{x - 1} = 5}$$

Question 8

Evaluate the limit:

$$\lim_{x \rightarrow 2^+} \frac{1}{x - 2}$$

Question 8 - Solution

We are given the limit:

$$\lim_{x \rightarrow 2^+} \frac{1}{x-2}$$

Step 1: Analyze the expression near $x = 2^+$

As x approaches 2 from the right, the value of $x - 2$ becomes a very small positive number. Therefore, $\frac{1}{x-2}$ becomes a very large positive number.

$$x \rightarrow 2^+ \quad \Rightarrow \quad x - 2 \rightarrow 0^+ \quad \Rightarrow \quad \frac{1}{x-2} \rightarrow +\infty$$

Conclusion:

$$\boxed{\lim_{x \rightarrow 2^+} \frac{1}{x-2} = +\infty}$$

This indicates that the function grows without bound as x approaches 2 from the right.

Question 9

Evaluate the limit:

$$\lim_{x \rightarrow 0^-} \frac{2x + 1}{x}$$

Question 9 - Solution

We are asked to evaluate:

$$\lim_{x \rightarrow 0^-} \frac{2x + 1}{x}$$

Step 1: Behavior near zero from the left

As $x \rightarrow 0^-$, the numerator $2x + 1 \rightarrow 1$, since $2x \rightarrow 0^-$ and 1 is a constant.

The denominator $x \rightarrow 0^-$, which is a small negative number.

So the entire fraction behaves like:

$$\frac{2x + 1}{x} \approx \frac{1}{\text{tiny negative}} = -\infty$$

Step 2: More formal analysis (optional)

We can write:

$$\frac{2x + 1}{x} = 2 + \frac{1}{x}$$

As $x \rightarrow 0^-$, $\frac{1}{x} \rightarrow -\infty$, and the constant 2 becomes negligible in comparison.

Conclusion:

$\lim_{x \rightarrow 0^-} \frac{2x + 1}{x} = -\infty$

Question 10

Evaluate the limit:

$$\lim_{x \rightarrow 3^+} \frac{5}{x - 3}$$

Question 10 - Solution

We are given:

$$\lim_{x \rightarrow 3^+} \frac{5}{x-3}$$

Step 1: Analyze the denominator

As $x \rightarrow 3^+$, the value of x is slightly greater than 3, so $x - 3$ is a small positive number.

$$x - 3 \rightarrow 0^+ \Rightarrow \frac{1}{x-3} \rightarrow +\infty$$

Multiplying by 5, we have:

$$\frac{5}{x-3} \rightarrow +\infty$$

Conclusion:

$$\boxed{\lim_{x \rightarrow 3^+} \frac{5}{x-3} = +\infty}$$