

Question 1

Solve the following equation for x :

$$3^{2x-1} = 7^{x+2}$$

Instructions: Solve algebraically using logarithms and express your answer in exact form. Approximate the result to 3 decimal places.

Question 1 - Solution

Taking natural logarithms (both sides are positive) gives

$$(2x - 1) \ln 3 = (x + 2) \ln 7.$$

Collect the terms in x and divide by their nonzero coefficient:

$$x = \frac{\ln 3 + 2 \ln 7}{2 \ln 3 - \ln 7} \approx 19.857.$$

The logarithm is one-to-one, so solving this equivalent linear equation introduces no extraneous solutions. The exact expression verifies the original equation.

Question 2

Solve the following equation:

$$e^{2x} - 7e^x + 10 = 0$$

- (a) Solve algebraically for all real solutions.
- (b) Check each solution for validity.

Question 2 - Solution

(a) Let $y = e^x$. Then the equation becomes:

$$y^2 - 7y + 10 = 0 \Rightarrow (y - 5)(y - 2) = 0 \Rightarrow y = 5 \quad \text{or} \quad y = 2$$

Now solve for x :

$$e^x = 5 \Rightarrow x = \ln(5)$$

$$e^x = 2 \Rightarrow x = \ln(2)$$

(b) **Check domain/validity:**

Both $\ln(2)$ and $\ln(5)$ are defined for positive inputs, and since both values came from valid solutions for $y = e^x > 0$, no extraneous solutions.

Final Answer:

$x = \ln(2) \quad \text{or} \quad x = \ln(5)$

Question 3

Solve the following equation:

$$\log_3(x^2 - 4x) = 1 + \log_3(x - 2)$$

- (a) Solve the equation algebraically.
- (b) Identify any extraneous solutions.

Question 3 - Solution

(a) **Combine the logarithms:**

Move the constant:

$$\log_3(x^2 - 4x) - \log_3(x - 2) = 1$$

Apply the quotient rule:

$$\log_3\left(\frac{x^2 - 4x}{x - 2}\right) = 1$$

Factor the numerator:

$$x^2 - 4x = x(x - 4) \Rightarrow \frac{x(x - 4)}{x - 2}$$

So:

$$\log_3\left(\frac{x(x - 4)}{x - 2}\right) = 1 \Rightarrow \frac{x(x - 4)}{x - 2} = 3^1 = 3$$

Multiply both sides:

$$x(x - 4) = 3(x - 2) \Rightarrow x^2 - 4x = 3x - 6 \Rightarrow x^2 - 7x + 6 = 0 \Rightarrow (x - 6)(x - 1) = 0 \Rightarrow x = 6, 1$$

(b) **Check for extraneous solutions:**

Logarithms are only defined for positive arguments.

Check domain: $x^2 - 4x > 0 \Rightarrow x(x - 4) > 0 \Rightarrow x < 0$ or $x > 4$, $x - 2 > 0 \Rightarrow x > 2$

Only values that satisfy both: $x > 4$

So: $x = 6 \rightarrow$ valid, $x = 1 \rightarrow$ not valid (fails both domain conditions)

Final Answer:

$$\boxed{x = 6}$$

Question 4

Solve for x in the equation:

$$2^{3x+1} = 5^{x-2}$$

Give your answer in exact form using logarithms, and approximate the result to three decimal places.

Question 4 - Solution

Taking natural logarithms (both sides are positive) gives

$$(3x + 1) \ln 2 = (x - 2) \ln 5.$$

Collect the terms in x and divide by their nonzero coefficient:

$$x = \frac{-\ln 2 - 2 \ln 5}{3 \ln 2 - \ln 5} \approx -8.323.$$

The logarithm is one-to-one, so solving this equivalent linear equation introduces no extraneous solutions. The exact expression verifies the original equation.

Question 5

Solve the equation:

$$\ln(x - 1) + \ln(x + 1) = \ln(6)$$

- (a) Solve for x algebraically.
- (b) Identify and exclude any extraneous solutions.

Question 5 - Solution

(a) Combine the logarithmic expressions using properties:

$$\ln(x-1) + \ln(x+1) = \ln[(x-1)(x+1)] = \ln(x^2-1)$$

So the equation becomes:

$$\ln(x^2-1) = \ln(6) \Rightarrow x^2-1 = 6 \Rightarrow x^2 = 7 \Rightarrow x = \pm\sqrt{7}$$

(b) **Domain Check:**

We must have:

$$x-1 > 0 \Rightarrow x > 1 \quad \text{and} \quad x+1 > 0 \Rightarrow x > -1 \Rightarrow \text{Combined: } x > 1$$

So $x = \sqrt{7} \approx 2.6458$ is valid, but $x = -\sqrt{7} \approx -2.6458$ is not.

$$\text{Final Answer: } \boxed{x = \sqrt{7}}$$

Question 6

Solve the equation:

$$4e^{2x} - 12e^x + 5 = 0$$

- (a) Solve algebraically for all real solutions.
- (b) Provide both exact and approximate answers.

Question 6 - Solution

(a) Let $y = e^x$. Then the equation becomes:

$$4y^2 - 12y + 5 = 0$$

Use the quadratic formula:

$$y = \frac{12 \pm \sqrt{(-12)^2 - 4(4)(5)}}{2(4)} = \frac{12 \pm \sqrt{144 - 80}}{8} = \frac{12 \pm \sqrt{64}}{8} = \frac{12 \pm 8}{8}$$

So:

$$y = \frac{20}{8} = 2.5 \quad \text{or} \quad y = \frac{4}{8} = 0.5$$

Now solve for x :

$$e^x = 2.5 \Rightarrow x = \ln(2.5), \quad e^x = 0.5 \Rightarrow x = \ln(0.5)$$

(b) Final Answers:

Exact:

$$\boxed{x = \ln(2.5) \quad \text{or} \quad x = \ln(0.5)}$$

Approximate:

$$x \approx \ln(2.5) \approx 0.9163, \quad x \approx \ln(0.5) \approx -0.6931$$

Question 7

Solve for all real solutions:

$$\log_2(x + 3) + \log_2(x - 1) = 3$$

Instructions: Solve algebraically and check for extraneous solutions.

Question 7 - Solution

We are given:

$$\log_2(x+3) + \log_2(x-1) = 3$$

Step 1: Combine the logs using the product rule:

$$\log_2[(x+3)(x-1)] = 3$$

Step 2: Convert from logarithmic to exponential form:

$$(x+3)(x-1) = 2^3 = 8 \Rightarrow x^2 + 2x - 3 = 8 \Rightarrow x^2 + 2x - 11 = 0$$

Step 3: Solve the quadratic:

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-11)}}{2(1)} = \frac{-2 \pm \sqrt{4+44}}{2} = \frac{-2 \pm \sqrt{48}}{2} = \frac{-2 \pm 4\sqrt{3}}{2} = \boxed{-1 \pm 2\sqrt{3}}$$

Step 4: Domain check:

We need:

$$x+3 > 0 \Rightarrow x > -3, \quad x-1 > 0 \Rightarrow x > 1 \Rightarrow \text{So } x > 1$$

Now test the two roots: $x = -1+2\sqrt{3} \approx -1+3.464 \approx 2.464 \rightarrow \text{valid}$, $x = -1-2\sqrt{3} \approx -1-3.464 \approx -4.464 \rightarrow \text{invalid}$

Final Answer:

$$\boxed{x = -1 + 2\sqrt{3}}$$

Question 8

Solve the equation:

$$5^{x+1} = 2^{3x-2}$$

Instructions: Solve the equation algebraically using logarithms. Provide both an exact expression and an approximate value rounded to 3 decimal places.

Question 8 - Solution

Taking natural logarithms (both sides are positive) gives

$$(x + 1) \ln 5 = (3x - 2) \ln 2.$$

Collect the terms in x and divide by their nonzero coefficient:

$$x = \frac{-\ln 5 - 2 \ln 2}{\ln 5 - 3 \ln 2} \approx 6.374.$$

The logarithm is one-to-one, so solving this equivalent linear equation introduces no extraneous solutions. The exact expression verifies the original equation.

Question 9

Solve for all real solutions:

$$\log(x^2 + 1) = 1 + \log(x - 2)$$

Instructions: Solve algebraically and check for extraneous solutions.

Question 9 - Solution

We are given:

$$\log(x^2 + 1) = 1 + \log(x - 2)$$

Step 1: Use properties of logarithms.

Subtract $\log(x - 2)$ from both sides:

$$\log(x^2 + 1) - \log(x - 2) = 1$$

Apply the quotient rule:

$$\log\left(\frac{x^2 + 1}{x - 2}\right) = 1$$

Step 2: Convert to exponential form.

Recall that $\log_b(A) = C \Rightarrow A = b^C$. Since no base is written, it is base 10:

$$\frac{x^2 + 1}{x - 2} = 10^1 = 10$$

$$x^2 + 1 = 10(x - 2) \Rightarrow x^2 + 1 = 10x - 20 \Rightarrow x^2 - 10x + 21 = 0 \Rightarrow (x - 3)(x - 7) = 0 \Rightarrow x = 3 \text{ or } x = 7$$

Step 3: Domain check.

We need: $x - 2 > 0 \Rightarrow x > 2$

Both $x = 3$ and $x = 7$ satisfy the domain.

Final Answer:

$x = 3 \quad \text{or} \quad x = 7$

Question 10

Solve the equation:

$$2^{x+1} + 2^x = 48$$

Instructions: Solve the equation algebraically. Express your answer in exact form and give an approximation rounded to three decimal places.

Question 10 - Solution

We are given:

$$2^{x+1} + 2^x = 48$$

Step 1: Use the property $2^{x+1} = 2 \cdot 2^x$:

$$2 \cdot 2^x + 2^x = 48 \Rightarrow 3 \cdot 2^x = 48 \Rightarrow 2^x = \frac{48}{3} = 16$$

Step 2: Solve for x :

$$2^x = 16 = 2^4 \Rightarrow \boxed{x = 4}$$

Exact answer: $\boxed{x = 4}$

Approximate answer: $\boxed{x = 4.000}$