

Question 1

The population of a bacterial culture is modeled by the function

$$P(t) = 500e^{0.3t}$$

where t is measured in hours.

- (a) What is the population after 5 hours?
- (b) After how many hours will the population reach 3000?
- (c) By what factor does the population grow every 4 hours?

Round your answers to two decimal places.

Question 1 - Solution

Use the exact exponential model and keep full precision until the final rounding.

(a)

$$P(5) = 500e^{1.5}$$

2240.84 bacteria

(b)

$$500e^{0.3t} = 3000 \implies t = \ln(6)/0.3$$

5.97 hours

(c)

$$P(t+4)/P(t) = e^{1.2}$$

3.32 growth factor

Question 2

A radioactive substance decays according to the formula

$$A(t) = A_0 e^{-kt}$$

where $A_0 = 100$ grams and the half-life of the substance is 10 hours.

- (a) Find the decay constant k .
- (b) How much of the substance remains after 24 hours?
- (c) After how many hours will only 20 grams remain?

Round all answers to two decimal places.

Question 2 - Solution

Use the exact exponential model and keep full precision until the final rounding.

(a)

$$e^{-10k} = 1/2 \implies k = \ln(2)/10$$

$$\boxed{0.07 \text{ hr}^{-1}}$$

The exact value of k above is used in later parts; its decimal value is 0.069315.

(b)

$$A(24) = 100e^{-24\ln(2)/10}$$

$$\boxed{18.95 \text{ grams}}$$

(c)

$$100e^{-kt} = 20 \implies t = 10 \ln(5)/\ln(2)$$

$$\boxed{23.22 \text{ hours}}$$

Question 3

A cup of coffee is initially at 190°F and is left to cool in a room that is maintained at 70°F. After 15 minutes, the coffee has cooled to 130°F.

Assume the temperature of the coffee follows Newton's Law of Cooling:

$$T(t) = T_{\text{room}} + (T_0 - T_{\text{room}})e^{-kt}$$

- (a) Find the value of the constant k .
- (b) What will the temperature be after 30 minutes?
- (c) After how many minutes will the coffee reach 100°F?

Round all answers to two decimal places.

Question 3 - Solution

Use the exact exponential model and keep full precision until the final rounding.

(a)

$$130 = 70 + 120e^{-15k} \implies k = \ln(2)/15$$

$$\boxed{0.05 \text{ min}^{-1}}$$

The exact value of k above is used in later parts; its decimal value is 0.046210.

(b)

$$T(30) = 70 + 120e^{-2 \ln 2}$$

$$\boxed{100.00 \text{ }^{\circ}\text{F}}$$

(c)

$$100 = 70 + 120e^{-kt} \implies t = \ln(4)/k$$

$$\boxed{30.00 \text{ minutes}}$$

Question 4

The value of a car decreases exponentially over time according to the function

$$V(t) = 25000e^{-0.18t}$$

where $V(t)$ is the value in dollars and t is the number of years since purchase.

- (a) What is the value of the car after 3 years?
- (b) How long will it take for the car to be worth half its original value?
- (c) After how many years will the car be worth \$5,000?

Question 4 - Solution

Use the exact exponential model and keep full precision until the final rounding.

(a)

$$V(3) = 25000e^{-0.54}$$

$$\boxed{14568.71 \text{ dollars}}$$

(b)

$$e^{-0.18t} = 1/2 \implies t = \ln(2)/0.18$$

$$\boxed{3.85 \text{ years}}$$

(c)

$$25000e^{-0.18t} = 5000 \implies t = \ln(5)/0.18$$

$$\boxed{8.94 \text{ years}}$$

Question 5

A scientist is growing a bacteria culture in a lab. The population of bacteria at time t hours is given by the function:

$$P(t) = 1200e^{0.55t}$$

- (a) What is the population after 4 hours?
- (b) After how many hours will the population reach 10,000?
- (c) By what percentage does the population increase each hour?

Round all answers to two decimal places.

Question 5 - Solution

Use the exact exponential model and keep full precision until the final rounding.

(a)

$$P(4) = 1200e^{2.2}$$

10830.02 bacteria

(b)

$$t = \ln(10000/1200)/0.55$$

3.86 hours

(c)

$$100(P(t+1)/P(t) - 1) = 100(e^{0.55} - 1)$$

73.33 %

Question 6

A forest fire is spreading in a circular pattern and the area (in square kilometers) covered by the fire after t hours is modeled by the function:

$$A(t) = 2.5e^{0.4t}$$

- (a) What is the area affected after 6 hours?
- (b) After how many hours will the fire have covered 50 square kilometers?
- (c) How fast is the area growing (in km^2/hr) after 6 hours?

Round all answers to two decimal places.

Question 6 - Solution

Use the exact exponential model and keep full precision until the final rounding.

(a)

$$A(6) = 2.5e^{2.4}$$

$$\boxed{27.56 \text{ km}^2}$$

(b)

$$t = \ln(50/2.5)/0.4$$

$$\boxed{7.49 \text{ hours}}$$

(c)

$$A'(t) = e^{0.4t}, \quad A'(6) = e^{2.4}$$

$$\boxed{11.02 \text{ km}^2/\text{hr}}$$

Question 7

The number of viewers watching a viral video on a social media platform can be modeled by the function

$$V(t) = 1500 \cdot e^{0.7t}$$

where $V(t)$ is the number of viewers and t is the time in days since the video was posted.

- (a) How many people have viewed the video after 2 days?
- (b) After how many days will the video have 50,000 viewers?
- (c) What is the average rate of change of viewers between day 1 and day 3?

Round all answers to two decimal places.

Question 7 - Solution

Use the exact exponential model and keep full precision until the final rounding.

(a)

$$V(2) = 1500e^{1.4}$$

$$\boxed{6082.80 \text{ viewers}}$$

(b)

$$t = \ln(50000/1500)/0.7$$

$$\boxed{5.01 \text{ days}}$$

(c)

$$\frac{V(3) - V(1)}{3 - 1} = 750(e^{2.1} - e^{0.7})$$

$$\boxed{4614.31 \text{ viewers/day}}$$

Question 8

A certain medication is administered intravenously and metabolized by the body over time. The amount of the drug in the bloodstream (in milligrams) is modeled by the function:

$$C(t) = 80e^{-0.35t}$$

where t is the number of hours after the drug is administered.

- (a) What is the amount of the drug in the bloodstream after 2 hours?
- (b) After how many hours will only 10 mg remain in the bloodstream?
- (c) What percentage of the drug is lost every hour?

Round all answers to two decimal places.

Question 8 - Solution

Use the exact exponential model and keep full precision until the final rounding.

(a)

$$C(2) = 80e^{-0.7}$$

39.73 mg

(b)

$$t = \ln(80/10)/0.35$$

5.94 hours

(c)

$$100(1 - C(t+1)/C(t)) = 100(1 - e^{-0.35})$$

29.53 %

Question 9

A savings account earns interest compounded continuously at an annual rate of 4.5

$$A(t) = 3000e^{0.045t}$$

- (a) How much money will be in the account after 6 years?
- (b) How long will it take for the account to reach \$5,000?
- (c) How much interest is earned between years 4 and 8?

Round all answers to two decimal places.

Question 9 - Solution

Use the exact exponential model and keep full precision until the final rounding.

(a)

$$A(6) = 3000e^{0.27}$$

3929.89 dollars

(b)

$$t = \ln(5000/3000)/0.045$$

11.35 years

(c)

$$A(8) - A(4) = 3000(e^{0.36} - e^{0.18})$$

708.34 dollars

Question 10

A certain species of insect is introduced to a new habitat. Its population is modeled by the exponential function:

$$P(t) = 500 \cdot e^{0.6t}$$

where $P(t)$ is the population size after t weeks.

- (a) What will the population be after 5 weeks?
- (b) After how many weeks will the population exceed 10,000?
- (c) How fast is the population growing at $t = 5$ weeks?

Round all answers to two decimal places.

Question 10 - Solution

Use the exact exponential model and keep full precision until the final rounding.

(a)

$$P(5) = 500e^3$$

10042.77 insects

(b)

$$t_* = \ln(10000/500)/0.6$$

4.99 weeks

(c)

$$P'(t) = 300e^{0.6t}, \quad P'(5) = 300e^3$$

6025.66 insects/week

The population equals 10000 at $t = t_*$ and exceeds it precisely when $t > t_*$. The rounded threshold is not an exact inequality boundary.