

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 1

Solve the equation

$$2 \cos(x) + 1 = 0$$

for all $x \in [0, 2\pi]$, and round your answers to two decimal places if necessary.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 1 - Solution

Step 1: Isolate the cosine term

$$2 \cos(x) + 1 = 0 \Rightarrow \cos(x) = -\frac{1}{2}$$

Step 2: Solve the equation using inverse cosine

$$\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} \Rightarrow \text{Reference angle} = \frac{\pi}{3}$$

Since cosine is negative, the solutions are in Quadrants II and III:

- Quadrant II: $x = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$ - Quadrant III: $x = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3}$$

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 2

Solve the equation

$$\sin(2x) = 0.6$$

for all $x \in [0, 2\pi]$, and round your answers to two decimal places.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 2 - Solution

Step 1: Isolate the trigonometric function. The equation becomes

$$\sin(2x) = 0.6, \quad u = 2x, \quad 0 \leq u \leq 4\pi.$$

Let $\alpha = \arcsin(0.6)$. All solutions are

$$u = \alpha + 2k\pi \quad \text{or} \quad u = \pi - \alpha + 2k\pi, \quad k \in \mathbb{Z}.$$

Step 2: Restrict and convert. Keep precisely the values of u in $[0, 4\pi]$ and divide by 2. Evaluating the inverse function at full precision and rounding only the final values gives

$$x \approx 0.32, 1.25, 3.46, 4.39.$$

These are all 4 solutions in the stated interval, in radians. Substitution of the unrounded values verifies the original equation.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 3

Solve the equation

$$\tan(2x) = -1.7$$

for all $x \in [0, 2\pi]$, and round your answers to two decimal places.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 3 - Solution

Step 1: Isolate the trigonometric function. The equation becomes

$$\tan(2x) = -1.7, \quad u = 2x, \quad 0 \leq u \leq 4\pi.$$

Let $\alpha = \arctan(-1.7)$. All solutions are

$$u = \alpha + k\pi, \quad k \in \mathbb{Z}.$$

Step 2: Restrict and convert. Keep precisely the values of u in $[0, 4\pi]$ and divide by 2. Evaluating the inverse function at full precision and rounding only the final values gives

$$x \approx 1.05, 2.62, 4.19, 5.76.$$

These are all 4 solutions in the stated interval, in radians. Substitution of the unrounded values verifies the original equation.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 4

Solve the equation

$$\cos(3x) = 0.2$$

for all $x \in [0, 2\pi]$, and round your answers to two decimal places.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 4 - Solution

Step 1: Isolate the trigonometric function. The equation becomes

$$\cos(3x) = 0.2, \quad u = 3x, \quad 0 \leq u \leq 6\pi.$$

Let $\alpha = \arccos(0.2)$. All solutions are

$$u = \pm\alpha + 2k\pi, \quad k \in \mathbb{Z}.$$

Step 2: Restrict and convert. Keep precisely the values of u in $[0, 6\pi]$ and divide by 3. Evaluating the inverse function at full precision and rounding only the final values gives

$$x \approx 0.46, 1.64, 2.55, 3.73, 4.65, 5.83.$$

These are all 6 solutions in the stated interval, in radians. Substitution of the unrounded values verifies the original equation.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 5

Solve the equation

$$2\sin(x) - 1 = 0.3$$

for all $x \in [0, 2\pi]$, and round your answers to two decimal places.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 5 - Solution

Step 1: Isolate the trigonometric function. The equation becomes

$$\sin(1x) = 0.65, \quad u = 1x, \quad 0 \leq u \leq 2\pi.$$

Let $\alpha = \arcsin(0.65)$. All solutions are

$$u = \alpha + 2k\pi \quad \text{or} \quad u = \pi - \alpha + 2k\pi, \quad k \in \mathbb{Z}.$$

Step 2: Restrict and convert. Keep precisely the values of u in $[0, 2\pi]$ and divide by 1. Evaluating the inverse function at full precision and rounding only the final values gives

$$x \approx 0.71, 2.43.$$

These are all 2 solutions in the stated interval, in radians. Substitution of the unrounded values verifies the original equation.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 6

Solve the equation

$$1 + 3 \cos(x) = 0.4$$

for all $x \in [0, 2\pi]$, and round your answers to two decimal places.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 6 - Solution

Step 1: Isolate the trigonometric function. The equation becomes

$$\cos(1x) = -0.2, \quad u = 1x, \quad 0 \leq u \leq 2\pi.$$

Let $\alpha = \arccos(-0.2)$. All solutions are

$$u = \pm\alpha + 2k\pi, \quad k \in \mathbb{Z}.$$

Step 2: Restrict and convert. Keep precisely the values of u in $[0, 2\pi]$ and divide by 1. Evaluating the inverse function at full precision and rounding only the final values gives

$$x \approx 1.77, 4.51.$$

These are all 2 solutions in the stated interval, in radians. Substitution of the unrounded values verifies the original equation.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 7

Solve the equation

$$4 \sin(2x) = 3$$

for all $x \in [0, 2\pi]$, and round your answers to two decimal places.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 7 - Solution

Step 1: Isolate the trigonometric function. The equation becomes

$$\sin(2x) = 0.75, \quad u = 2x, \quad 0 \leq u \leq 4\pi.$$

Let $\alpha = \arcsin(0.75)$. All solutions are

$$u = \alpha + 2k\pi \quad \text{or} \quad u = \pi - \alpha + 2k\pi, \quad k \in \mathbb{Z}.$$

Step 2: Restrict and convert. Keep precisely the values of u in $[0, 4\pi]$ and divide by 2. Evaluating the inverse function at full precision and rounding only the final values gives

$$x \approx 0.42, 1.15, 3.57, 4.29.$$

These are all 4 solutions in the stated interval, in radians. Substitution of the unrounded values verifies the original equation.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 8

Solve the equation

$$2 \cos(2x) - 1 = 0.6$$

for all $x \in [0, 2\pi]$, and round your answers to two decimal places.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 8 - Solution

Step 1: Isolate the trigonometric function. The equation becomes

$$\cos(2x) = 0.8, \quad u = 2x, \quad 0 \leq u \leq 4\pi.$$

Let $\alpha = \arccos(0.8)$. All solutions are

$$u = \pm\alpha + 2k\pi, \quad k \in \mathbb{Z}.$$

Step 2: Restrict and convert. Keep precisely the values of u in $[0, 4\pi]$ and divide by 2. Evaluating the inverse function at full precision and rounding only the final values gives

$$x \approx 0.32, 2.82, 3.46, 5.96.$$

These are all 4 solutions in the stated interval, in radians. Substitution of the unrounded values verifies the original equation.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 9

Solve the equation

$$\tan(x) + 2 = 1.5$$

for all $x \in [0, 2\pi]$, and round your answers to two decimal places.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 9 - Solution

Step 1: Isolate the trigonometric function. The equation becomes

$$\tan(1x) = -0.5, \quad u = 1x, \quad 0 \leq u \leq 2\pi.$$

Let $\alpha = \arctan(-0.5)$. All solutions are

$$u = \alpha + k\pi, \quad k \in \mathbb{Z}.$$

Step 2: Restrict and convert. Keep precisely the values of u in $[0, 2\pi]$ and divide by 1. Evaluating the inverse function at full precision and rounding only the final values gives

$$x \approx 2.68, 5.82.$$

These are all 2 solutions in the stated interval, in radians. Substitution of the unrounded values verifies the original equation.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 10

Solve the equation

$$\sin(x) + \cos(x) = 1.2$$

for all $x \in [0, 2\pi]$, rounding your answers to two decimal places.

Calculus I Section 1-5: Solving Trig Equations with Calculators II

Question 10 - Solution

Use

$$\sin x + \cos x = \sqrt{2} \sin(x + \pi/4).$$

Let $\alpha = \arcsin(1.2/\sqrt{2})$. The two solution families are

$$x = \alpha - \pi/4 + 2k\pi, \quad x = 3\pi/4 - \alpha + 2k\pi.$$

Only $k = 0$ gives values in $[0, 2\pi]$, hence

$$x \approx 0.23, 1.34.$$

Both satisfy the unsquared equation; adding π reverses the sign and does not give another solution.