

Question 1

Sketch the graph of the function below. Identify the domain, range, intercepts, asymptotes, and any transformations applied to a basic parent function.

$$f(x) = -\frac{1}{x-2} + 3$$

Instructions: Analyze the function using transformations of the reciprocal function $f(x) = \frac{1}{x}$.

Question 1 - Solution

We are given:

$$f(x) = -\frac{1}{x-2} + 3$$

Parent Function: $\frac{1}{x}$

Transformations:

- Horizontal shift right by 2
- Reflection over x-axis (due to the negative sign)
- Vertical shift up by 3

Domain: $(-\infty, 2) \cup (2, \infty)$

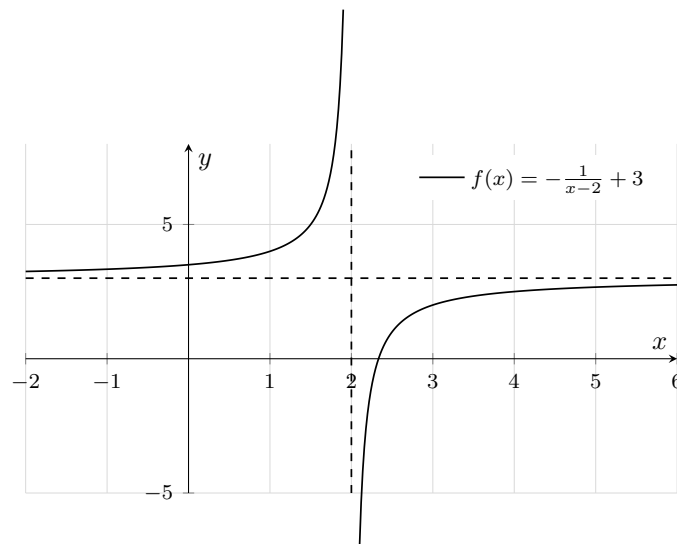
Range: $(-\infty, 3) \cup (3, \infty)$

Vertical Asymptote: $x = 2$

Horizontal Asymptote: $y = 3$

Y-intercept: $f(0) = \frac{7}{2}$

X-intercept: $f(x) = 0 \Rightarrow x = \frac{7}{3}$



Question 2

Sketch the graph of the function:

$$f(x) = |x + 1| - 2$$

Instructions: Identify the vertex, axis of symmetry, domain, range, and x- and y-intercepts. Then sketch the graph.

Question 2 - Solution

We are given:

$$f(x) = |x + 1| - 2$$

Step 1: Transform the parent function $f(x) = |x|$

- Shift left by 1: $x \rightarrow x + 1$
- Shift down by 2: $f(x) \rightarrow f(x) - 2$

Vertex: The vertex of $f(x) = |x|$ is at $(0, 0)$, so for this function it's at $(-1, -2)$

Axis of Symmetry: $x = -1$

Domain: $(-\infty, \infty)$

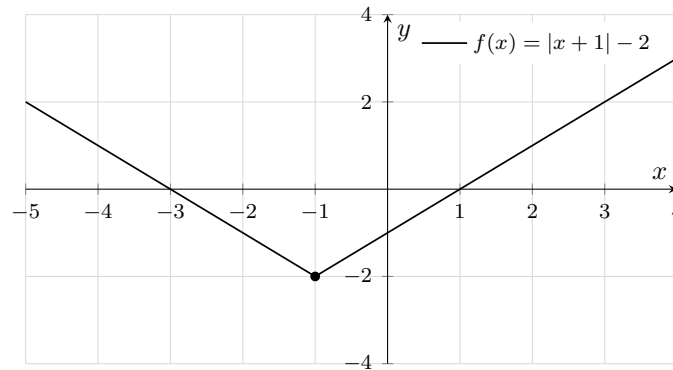
Range: $[-2, \infty)$

X-intercepts:

$$|x + 1| - 2 = 0 \Rightarrow |x + 1| = 2 \Rightarrow x + 1 = \pm 2 \Rightarrow x = 1, -3$$

Y-intercept:

$$f(0) = |0 + 1| - 2 = 1 - 2 = -1$$



Question 3

Sketch the graph of the piecewise function:

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 0 \\ 2x + 1 & \text{if } x \geq 0 \end{cases}$$

Instructions: Clearly identify continuity, domain, range, intercepts, and sketch the graph.

Question 3 - Solution

We are given a piecewise function:

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 0 \\ 2x + 1 & \text{if } x \geq 0 \end{cases}$$

Step 1: Domain

Both pieces are defined on their respective intervals, so:

$$\text{Domain: } (-\infty, \infty)$$

Step 2: Range

For $x < 0$: $f(x) = x^2 + 1 \Rightarrow f(x) > 1$, For $x \geq 0$: $f(x) = 2x + 1 \Rightarrow f(x) \geq 1$

So the function reaches its minimum at $f(0) = 1$, and increases in both directions:

$$\text{Range: } [1, \infty)$$

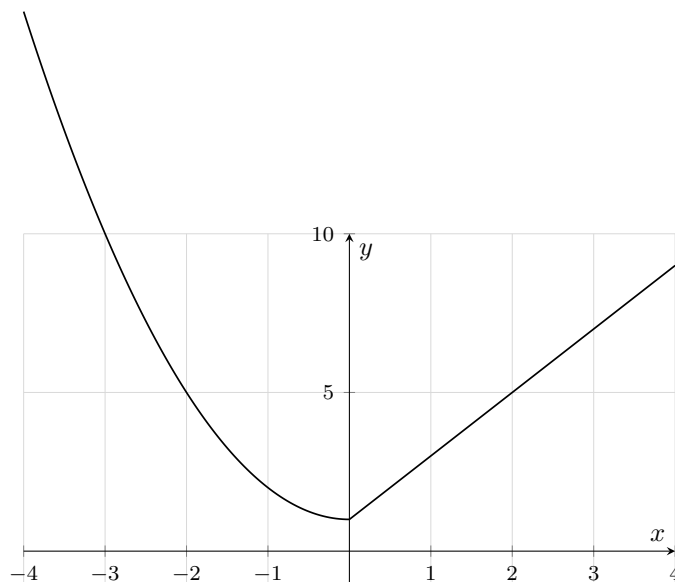
Step 3: Continuity at $x = 0$

$$\lim_{x \rightarrow 0^-} f(x) = 0^2 + 1 = 1, \quad \lim_{x \rightarrow 0^+} f(x) = 2(0) + 1 = 1 \Rightarrow \text{Function is continuous at } x = 0$$

Step 4: Intercepts

Y-intercept: $f(0) = 1$ **X-intercept:** Solve $f(x) = 0$: , $x^2 + 1 = 0 \Rightarrow$ no real solution , $2x + 1 = 0 \Rightarrow x = -\frac{1}{2} \notin [0, \infty) \Rightarrow$ no x-intercept

Conclusion: Domain: $(-\infty, \infty)$, Range: $[1, \infty)$, No x-intercepts, y-intercept at $(0, 1)$, Continuous everywhere



$$f(x) = \begin{cases} x^2 + 1 & x < 0 \\ 2x + 1 & x \geq 0 \end{cases}$$

Question 4

Sketch the graph of the function:

$$f(x) = \sqrt{x+2} - 1$$

Instructions: Identify the domain, range, intercepts, and transformations applied to the parent square root function. Then provide a sketch.

Question 4 - Solution

We are given:

$$f(x) = \sqrt{x+2} - 1$$

Step 1: Parent function: $f(x) = \sqrt{x}$

Transformations:

- Horizontal shift left by 2: $x \rightarrow x + 2$
- Vertical shift down by 1

Step 2: Domain:

$$x + 2 \geq 0 \Rightarrow x \geq -2 \Rightarrow \boxed{\text{Domain: } [-2, \infty)}$$

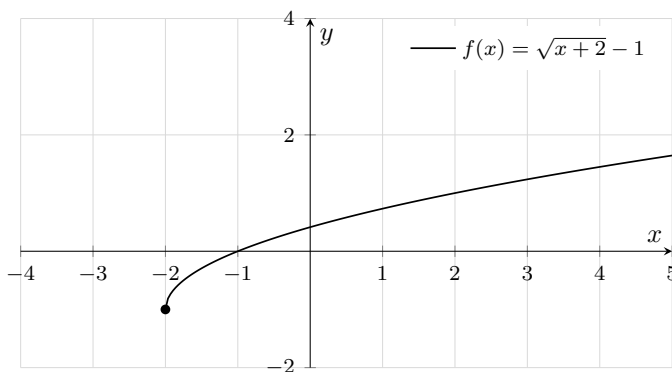
Step 3: Range: Since square root outputs ≥ 0 , and we subtract 1:

$$\boxed{\text{Range: } [-1, \infty)}$$

Step 4: Intercepts

- **Y-intercept:** $f(0) = \sqrt{2} - 1 \approx 0.414$
- **X-intercept:** Set $f(x) = 0 \Rightarrow \sqrt{x+2} = 1 \Rightarrow x+2 = 1 \Rightarrow x = -1$

Key Point: The graph starts at $(-2, -1)$



Question 5

Sketch the graph of the function:

$$f(x) = -|x - 3| + 2$$

Instructions: Identify the vertex, axis of symmetry, domain, range, and intercepts. Then sketch the graph of the function using transformations.

Question 5 - Solution

We are given:

$$f(x) = -|x - 3| + 2$$

Step 1: Parent function: $f(x) = |x|$

Transformations:

- Shift right by 3: $x \rightarrow x - 3$
- Reflect over x-axis: negative sign
- Shift up by 2

Step 2: Vertex:

Vertex at $(3, 2)$

Step 3: Axis of symmetry:

$$x = 3$$

Step 4: Domain and Range

Domain: $(-\infty, \infty)$

Range: $(-\infty, 2]$

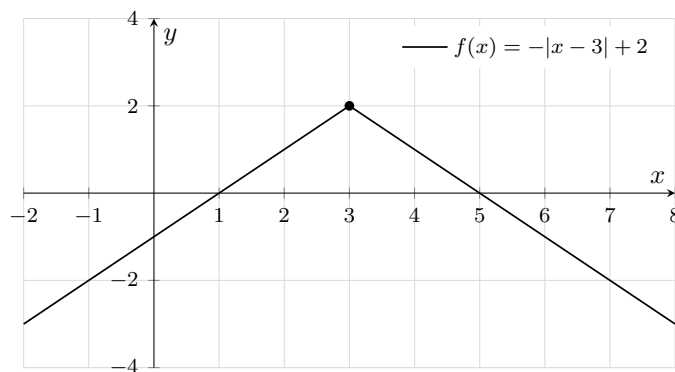
Step 5: Intercepts

- **Y-intercept:** Set $x = 0 \Rightarrow f(0) = -|0 - 3| + 2 = -3 + 2 = -1$ - **X-intercepts:** Solve $f(x) = 0$

$$-|x - 3| + 2 = 0 \Rightarrow |x - 3| = 2 \Rightarrow x - 3 = \pm 2 \Rightarrow x = 1, 5$$

Final Answer Summary:

- Vertex: $(3, 2)$
- Axis of symmetry: $x = 3$
- Domain: $(-\infty, \infty)$
- Range: $(-\infty, 2]$
- X-intercepts: $x = 1, 5$
- Y-intercept: $y = -1$



Question 6

Sketch the graph of the rational function:

$$f(x) = \frac{x^2 - 1}{x^2 - 4}$$

Instructions: Identify all intercepts, vertical and horizontal asymptotes, domain, and any removable discontinuities. Then sketch the graph based on your analysis.

Question 6 - Solution

We are given:

$$f(x) = \frac{x^2 - 1}{x^2 - 4} = \frac{(x-1)(x+1)}{(x-2)(x+2)}$$

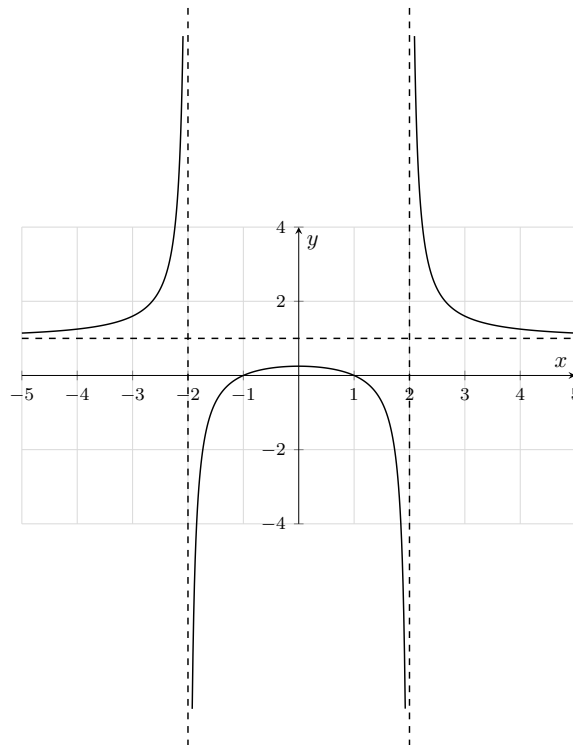
Domain: $x \neq \pm 2 \Rightarrow (-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

Vertical Asymptotes: $x = -2$ and $x = 2$

Horizontal Asymptote: Degrees of numerator and denominator are equal: $y = \frac{1}{1} = \boxed{y = 1}$

X-intercepts: Set numerator = 0: $x = \pm 1$

Y-intercept: $f(0) = \frac{-1}{-4} = \frac{1}{4}$



Question 7

Sketch the graph of the function:

$$f(x) = \frac{1}{x^2 + 1}$$

Instructions: Identify the domain, range, intercepts, end behavior, and any symmetry of the function. Then sketch the graph.

Question 7 - Solution

We are given:

$$f(x) = \frac{1}{x^2 + 1}$$

Step 1: Domain

Since $x^2 + 1 > 0$ for all real x , the domain is:

$$(-\infty, \infty)$$

Step 2: Y-intercept

Set $x = 0$:

$$f(0) = \frac{1}{0^2 + 1} = 1$$

Step 3: X-intercepts

Set $f(x) = 0$:

$$\frac{1}{x^2 + 1} = 0 \Rightarrow \text{no solution}$$

So, **no x-intercepts**.

Step 4: Range

Since the denominator is always ≥ 1 , the maximum occurs at $x = 0$, and the function decreases on both sides. Therefore:

$$\text{Range: } (0, 1]$$

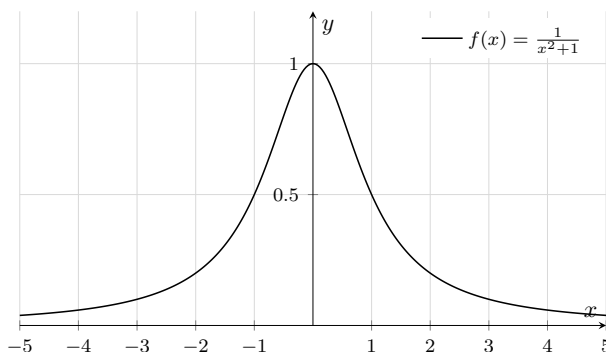
Step 5: Symmetry

$$f(-x) = \frac{1}{(-x)^2 + 1} = \frac{1}{x^2 + 1} = f(x)$$

So the function is **even**, symmetric about the y-axis.

Step 6: End Behavior

As $x \rightarrow \pm\infty$, $f(x) \rightarrow 0$



Question 8

Sketch the graph of the piecewise function:

$$f(x) = \begin{cases} x + 2 & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x \leq 2 \\ 4 - x & \text{if } x > 2 \end{cases}$$

Instructions: Clearly indicate open and closed points on the graph. Identify points of discontinuity, if any.

Question 8 - Solution

We are given a piecewise function:

$$f(x) = \begin{cases} x + 2 & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x \leq 2 \\ 4 - x & \text{if } x > 2 \end{cases}$$

Step 1: Evaluate endpoints to check continuity.

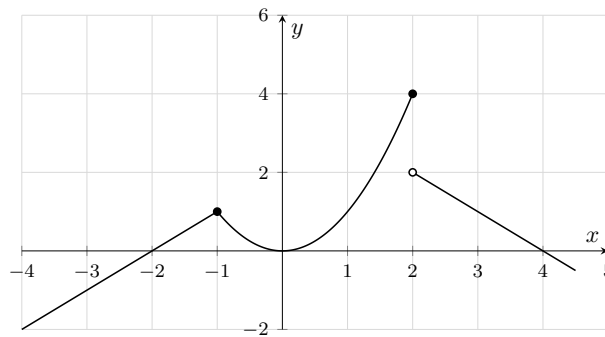
From the left of $x = -1$, $\lim_{x \rightarrow -1^-} f(x) = -1 + 2 = 1$

At $x = -1$, $f(-1) = (-1)^2 = 1 \Rightarrow$ continuous

From the right of $x = 2$, $\lim_{x \rightarrow 2^+} f(x) = 4 - 2 = 2$

At $x = 2$, $f(2) = 2^2 = 4 \Rightarrow$ discontinuous at $x = 2$

Discontinuity: Jump discontinuity at $x = 2$



Question 9

Sketch the graph of the function:

$$f(x) = \ln(x^2 + 1)$$

Instructions: Identify the domain, intercepts, end behavior, and any symmetry. Then sketch the graph.

Question 9 - Solution

We are given:

$$f(x) = \ln(x^2 + 1)$$

Domain: Since $x^2 + 1 > 0$ for all real x ,

Domain: $(-\infty, \infty)$

Y-intercept:

$$f(0) = \ln(0^2 + 1) = \ln(1) = 0$$

X-intercepts: Set $f(x) = 0$:

$$\ln(x^2 + 1) = 0 \Rightarrow x^2 + 1 = 1 \Rightarrow x^2 = 0 \Rightarrow x = 0$$

So the only intercept is at the origin.

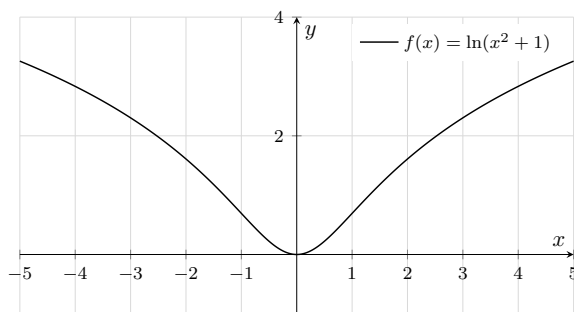
Symmetry:

$$f(-x) = \ln((-x)^2 + 1) = \ln(x^2 + 1) = f(x) \Rightarrow \text{even function}$$

End behavior: As $x \rightarrow \pm\infty$, $x^2 + 1 \rightarrow \infty \Rightarrow f(x) \rightarrow \infty$

Range: Minimum value is at $x = 0$: $\ln(1) = 0$, and it increases without bound.

Range: $[0, \infty)$



Question 10

Sketch the graph of the function:

$$f(x) = \frac{x}{\sqrt{x^2 + 1}}$$

Instructions: Identify the domain, range, intercepts, asymptotic behavior, and any symmetry. Then sketch the graph clearly.

Question 10 - Solution

We are given:

$$f(x) = \frac{x}{\sqrt{x^2 + 1}}$$

Step 1: Domain

The denominator is always positive and defined for all real x :

$$\text{Domain: } (-\infty, \infty)$$

Step 2: Intercepts

Y-intercept:

$$f(0) = \frac{0}{\sqrt{0^2 + 1}} = 0$$

X-intercept:

$$f(x) = 0 \Rightarrow \frac{x}{\sqrt{x^2 + 1}} = 0 \Rightarrow x = 0$$

Step 3: Symmetry

Check if the function is odd:

$$f(-x) = \frac{-x}{\sqrt{(-x)^2 + 1}} = \frac{-x}{\sqrt{x^2 + 1}} = -f(x)$$

So the function is **odd**, symmetric about the origin.

Step 4: End Behavior

As $x \rightarrow \pm\infty$:

$$f(x) = \frac{x}{\sqrt{x^2 + 1}} \rightarrow \pm 1$$

So horizontal asymptotes:

$$\boxed{y = 1} \quad \text{as } x \rightarrow \infty \quad \text{and} \quad \boxed{y = -1} \quad \text{as } x \rightarrow -\infty$$

Step 5: Range

$$\text{Range: } (-1, 1)$$

The function approaches 1 and -1 but never reaches them.

