

Question 10

Suppose that f is continuous on an interval I and that

$$\int f(x) dx = F(x) + C.$$

Show that the family of curves

$$y = F(x) + C$$

consists of vertical translations of a single curve.

Question 10 - Solution

Consider two members of the family of antiderivatives:

$$y_1 = F(x) + C_1, \quad y_2 = F(x) + C_2,$$

where C_1 and C_2 are constants.

Subtract the equations:

$$y_2 - y_1 = (F(x) + C_2) - (F(x) + C_1) = C_2 - C_1.$$

The difference $C_2 - C_1$ is a constant independent of x . Thus, for every value of x , the vertical distance between the two curves is the same.

This means that changing the constant of integration does not alter the shape of the graph, but only shifts it upward or downward by a fixed amount.

Therefore, the family of functions

$$y = F(x) + C$$

represents all vertical translations of a single curve.

Different values of C produce vertical shifts of the same graph