

Question 8

Suppose that f is continuous on an interval I and that

$$\int f(x) \, dx = F(x) + C.$$

Prove that if

$$F'(x) = 0 \quad \text{for all } x \in I,$$

then

$$\int f(x) \, dx = C.$$

Question 8 - Solution

Since

$$\int f(x) dx = F(x) + C,$$

the function $F(x)$ is an antiderivative of f .

Given that

$$F'(x) = 0 \quad \text{for all } x \in I,$$

it follows from the Mean Value Theorem that F is constant on the interval I .

Thus, there exists a real number K such that

$$F(x) = K \quad \text{for all } x \in I.$$

Substitute this into the expression for the indefinite integral:

$$\int f(x) dx = K + C.$$

Since $K + C$ is itself a constant, we may write

$$\int f(x) dx = C'.$$

Therefore, if the derivative of an antiderivative is zero on an interval, the indefinite integral reduces to a constant.

$$\boxed{\int f(x) dx = C}$$