

Question 7

Suppose that f is continuous on an interval I and that

$$\int_a^x f(t) dt = F(x) + C$$

for some constant C . Show that choosing a different lower limit $b \in I$ changes the antiderivative only by a constant.

Question 7 - Solution

Consider the two functions

$$F_1(x) = \int_a^x f(t) dt, \quad F_2(x) = \int_b^x f(t) dt.$$

Both F_1 and F_2 are antiderivatives of f on I by the Fundamental Theorem of Calculus.

Evaluate the difference between them:

$$F_1(x) - F_2(x) = \int_a^x f(t) dt - \int_b^x f(t) dt.$$

Using properties of definite integrals,

$$\int_a^x f(t) dt - \int_b^x f(t) dt = \int_a^b f(t) dt.$$

The right-hand side is a fixed real number that does not depend on x .

Therefore,

$$F_1(x) - F_2(x) = \text{constant}.$$

This shows that changing the lower limit of integration alters the antiderivative only by an additive constant.

$$\boxed{\int_a^x f(t) dt = \int_b^x f(t) dt + \text{constant}}$$