

Question 6

Let f be continuous on an interval I , and fix one antiderivative F . Write the general antiderivative as

$$y(x) = F(x) + C.$$

Show that the condition

$$y(x_0) = y_0, \quad x_0 \in I,$$

uniquely determines C .

Question 6 - Solution

Fix an antiderivative F of f on I . Every other antiderivative has the form $y(x) = F(x) + C$.

Imposing $y(x_0) = y_0$ yields $F(x_0) + C = y_0$, so

$$\boxed{C = y_0 - F(x_0).}$$

This value is unique because F , x_0 , and y_0 are fixed.