

Question 5

Suppose that f and g are continuous on a nondegenerate interval I , that $a \in I$, and that

$$\int_a^x f(t) \, dt = \int_a^x g(t) \, dt \quad \text{for all } x \in I.$$

Prove that

$$f(x) = g(x) \quad \text{for all } x \in I.$$

Question 5 - Solution

Set $F(x) = \int_a^x f(t) dt$ and $G(x) = \int_a^x g(t) dt$.

The hypothesis gives $F = G$ on I . At every interior point x of I , continuity of both integrands allows the Fundamental Theorem of Calculus:

$$f(x) = F'(x) = G'(x) = g(x).$$

At an included endpoint, take limits from the interior and use continuity of f and g . Thus

$$\boxed{f(x) = g(x) \quad \text{for every } x \in I.}$$