

Question 2

Suppose that $F'(x) = 0$ for all x in an interval I . Prove that F is constant on I .

Question 2 - Solution

Let $x_1, x_2 \in I$ with $x_1 < x_2$. We will show that

$$F(x_1) = F(x_2).$$

Since $F'(x) = 0$ for all $x \in I$, the function F is differentiable on I and continuous on the closed interval $[x_1, x_2]$.

By the Mean Value Theorem, there exists a number $c \in (x_1, x_2)$ such that

$$F'(c) = \frac{F(x_2) - F(x_1)}{x_2 - x_1}.$$

But $F'(c) = 0$ by assumption, so

$$0 = \frac{F(x_2) - F(x_1)}{x_2 - x_1}.$$

Since $x_2 - x_1 \neq 0$, it follows that

$$F(x_2) - F(x_1) = 0.$$

Thus,

$$F(x_2) = F(x_1).$$

Because x_1 and x_2 were arbitrary points in I , the function F has the same value at every point of I .

F is constant on I