

Question 1

Suppose that F and G are antiderivatives of the same function f on an interval I . Prove that there exists a constant C such that

$$F(x) = G(x) + C \quad \text{for all } x \in I.$$

Question 1 - Solution

Since F and G are both antiderivatives of f on I , we have

$$F'(x) = f(x) \quad \text{and} \quad G'(x) = f(x) \quad \text{for all } x \in I.$$

Consider the function

$$H(x) = F(x) - G(x).$$

Differentiate $H(x)$:

$$H'(x) = F'(x) - G'(x) = f(x) - f(x) = 0 \quad \text{for all } x \in I.$$

Thus, the derivative of H is zero on the interval I .

From the result that a function with zero derivative on an interval is constant, there exists a real number C such that

$$H(x) = C \quad \text{for all } x \in I.$$

Substituting back for $H(x)$ gives

$$F(x) - G(x) = C.$$

Rewriting,

$$F(x) = G(x) + C.$$

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