

Question 9

Prove that for every positive integer n ,

$$\sum_{k=1}^n \frac{k}{k+1} = n - \sum_{k=1}^n \frac{1}{k+1}.$$

Question 9 - Solution

For every $k \geq 1$,

$$\frac{k}{k+1} = 1 - \frac{1}{k+1}.$$

Summing all n terms gives

$$\boxed{\sum_{k=1}^n \frac{k}{k+1} = n - \sum_{k=1}^n \frac{1}{k+1}.$$

Equivalently, the result is $n + 1 - H_{n+1}$, where $H_{n+1} = \sum_{j=1}^{n+1} 1/j$.
For $n = 1$, both sides of the corrected identity equal $1/2$.