

Question 8

Prove that for any positive integer n and real number $r \neq 1$,

$$\sum_{k=0}^n r^k = \frac{1 - r^{n+1}}{1 - r}.$$

Question 8 - Solution

Let

$$S = \sum_{k=0}^n r^k = 1 + r + r^2 + \cdots + r^n.$$

Multiply both sides by r :

$$rS = r + r^2 + r^3 + \cdots + r^{n+1}.$$

Subtract the second equation from the first:

$$S - rS = (1 + r + r^2 + \cdots + r^n) - (r + r^2 + \cdots + r^{n+1}).$$

All intermediate terms cancel, leaving

$$S - rS = 1 - r^{n+1}.$$

Factor the left-hand side:

$$(1 - r)S = 1 - r^{n+1}.$$

Since $r \neq 1$, divide both sides by $1 - r$:

$$S = \frac{1 - r^{n+1}}{1 - r}.$$

Thus,

$$\boxed{\sum_{k=0}^n r^k = \frac{1 - r^{n+1}}{1 - r}}$$