

Question 7

Prove that for any positive integer n ,

$$\sum_{k=1}^n \frac{1}{k(k+1)} = \frac{n}{n+1}.$$

Question 7 - Solution

Begin by rewriting the general term using partial fractions.

For each $k \geq 1$,

$$\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}.$$

Substitute this into the sum:

$$\sum_{k=1}^n \frac{1}{k(k+1)} = \sum_{k=1}^n \left(\frac{1}{k} - \frac{1}{k+1} \right).$$

Write out the first few terms:

$$\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1}\right).$$

All intermediate terms cancel, leaving

$$1 - \frac{1}{n+1}.$$

Simplify:

$$1 - \frac{1}{n+1} = \frac{n}{n+1}.$$

$$\boxed{\sum_{k=1}^n \frac{1}{k(k+1)} = \frac{n}{n+1}}$$