

Question 6

Prove that for any positive integer n ,

$$\sum_{k=1}^n \left(\frac{1}{k} - \frac{1}{k+1} \right) = \frac{n}{n+1}.$$

Question 6 - Solution

Write out the first few terms of the sum:

$$\sum_{k=1}^n \left(\frac{1}{k} - \frac{1}{k+1} \right) = \left(1 - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \left(\frac{1}{3} - \frac{1}{4} \right) + \cdots + \left(\frac{1}{n} - \frac{1}{n+1} \right).$$

Notice that consecutive terms cancel:

$$-\frac{1}{2} + \frac{1}{2} = 0, \quad -\frac{1}{3} + \frac{1}{3} = 0, \quad \dots$$

After all cancellations, only the first and last terms remain:

$$1 - \frac{1}{n+1}.$$

Simplify:

$$1 - \frac{1}{n+1} = \frac{n}{n+1}.$$

$$\boxed{\sum_{k=1}^n \left(\frac{1}{k} - \frac{1}{k+1} \right) = \frac{n}{n+1}}$$