

Question 5

Prove that for any positive integer n ,

$$\sum_{k=1}^n (3k + 2) = \frac{3n(n+1)}{2} + 2n.$$

Question 5 - Solution

We prove the identity using properties of summations.

Start with the given sum:

$$\sum_{k=1}^n (3k + 2).$$

Split the sum using linearity:

$$\sum_{k=1}^n (3k + 2) = \sum_{k=1}^n 3k + \sum_{k=1}^n 2.$$

Factor out constants:

$$\sum_{k=1}^n 3k = 3 \sum_{k=1}^n k, \quad \sum_{k=1}^n 2 = 2 \sum_{k=1}^n 1.$$

Use known summation formulas:

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^n 1 = n.$$

Substitute these results:

$$\sum_{k=1}^n (3k + 2) = 3 \cdot \frac{n(n+1)}{2} + 2n.$$

Thus,

$$\sum_{k=1}^n (3k + 2) = \frac{3n(n+1)}{2} + 2n.$$

$$\sum_{k=1}^n (3k + 2) = \frac{3n(n+1)}{2} + 2n$$