

Question 1

Prove that for any positive integer n ,

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}.$$

Question 1 - Solution

We prove the formula using mathematical induction.

Base Case:

For $n = 1$,

$$\sum_{k=1}^1 k = 1, \quad \frac{1(1+1)}{2} = 1.$$

Thus, the formula holds for $n = 1$.

Inductive Hypothesis:

Assume that for some positive integer n ,

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}.$$

Inductive Step:

Consider the sum up to $n + 1$:

$$\sum_{k=1}^{n+1} k = \sum_{k=1}^n k + (n+1).$$

Using the inductive hypothesis,

$$\sum_{k=1}^{n+1} k = \frac{n(n+1)}{2} + (n+1).$$

Factor out $(n+1)$:

$$\sum_{k=1}^{n+1} k = (n+1) \left(\frac{n}{2} + 1 \right) = (n+1) \frac{n+2}{2}.$$

Thus,

$$\sum_{k=1}^{n+1} k = \frac{(n+1)(n+2)}{2}.$$

This matches the given formula with n replaced by $n + 1$.

By mathematical induction, the formula holds for all positive integers n .

$$\boxed{\sum_{k=1}^n k = \frac{n(n+1)}{2}}$$