

Question 9

Prove that

$$\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}} = 0.$$

Question 9 - Solution

We compare the growth rates of the logarithmic function and the power function.

Both $\ln x$ and $\sqrt{x}$ are positive and differentiable for $x > 0$. Apply L'Hôpital's Rule to the limit

$$\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}}.$$

Differentiate the numerator and denominator:

$$\frac{d}{dx}(\ln x) = \frac{1}{x}, \quad \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}.$$

Thus,

$$\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = \lim_{x \rightarrow \infty} \frac{2\sqrt{x}}{x}.$$

Simplify:

$$\frac{2\sqrt{x}}{x} = \frac{2}{\sqrt{x}}.$$

Since

$$\lim_{x \rightarrow \infty} \frac{1}{\sqrt{x}} = 0,$$

it follows that

$$\lim_{x \rightarrow \infty} \frac{2}{\sqrt{x}} = 0.$$

Therefore,

$$\lim_{x \rightarrow \infty} \frac{\ln x}{\sqrt{x}} = 0.$$

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