

Question 8

Prove that

$$\lim_{x \rightarrow \infty} (\ln(x+1) - \ln x) = 0.$$

Question 8 - Solution

Combine the logarithms using logarithmic properties:

$$\ln(x+1) - \ln x = \ln\left(\frac{x+1}{x}\right).$$

Rewrite the fraction:

$$\frac{x+1}{x} = 1 + \frac{1}{x}.$$

Thus, the expression becomes

$$\ln\left(1 + \frac{1}{x}\right).$$

Now take the limit as $x \rightarrow \infty$. Since

$$\lim_{x \rightarrow \infty} \frac{1}{x} = 0,$$

and the natural logarithm function is continuous at 1, we obtain

$$\lim_{x \rightarrow \infty} \ln\left(1 + \frac{1}{x}\right) = \ln(1) = 0.$$

$$\boxed{0}$$