

Question 4

Prove that

$$\lim_{x \rightarrow \infty} (x - \sqrt{x^2 - 3x}) = \frac{3}{2}.$$

Question 4 - Solution

We simplify the expression using algebraic manipulation.

Consider

$$x - \sqrt{x^2 - 3x}.$$

Multiply and divide by the conjugate:

$$x - \sqrt{x^2 - 3x} = \frac{(x - \sqrt{x^2 - 3x})(x + \sqrt{x^2 - 3x})}{x + \sqrt{x^2 - 3x}}.$$

Simplify the numerator:

$$x^2 - (x^2 - 3x) = 3x.$$

Thus,

$$x - \sqrt{x^2 - 3x} = \frac{3x}{x + \sqrt{x^2 - 3x}}.$$

Factor x out of the square root in the denominator:

$$\sqrt{x^2 - 3x} = x\sqrt{1 - \frac{3}{x}}.$$

Substitute into the expression:

$$\frac{3x}{x + x\sqrt{1 - \frac{3}{x}}} = \frac{3}{1 + \sqrt{1 - \frac{3}{x}}}.$$

Now take the limit as $x \rightarrow \infty$:

$$\lim_{x \rightarrow \infty} \frac{3}{1 + \sqrt{1 - \frac{3}{x}}} = \frac{3}{1 + \sqrt{1}} = \frac{3}{2}.$$

$$\boxed{\frac{3}{2}}$$