

Question 1

Prove that

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x} = 0.$$

Question 1 - Solution

We compare the growth rates of the numerator and denominator.

For $x > 0$, both $\ln x$ and x are differentiable. Apply L'Hôpital's Rule to the limit

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x}.$$

Differentiate the numerator and denominator:

$$\frac{d}{dx}(\ln x) = \frac{1}{x}, \quad \frac{d}{dx}(x) = 1.$$

Thus,

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x} = \lim_{x \rightarrow \infty} \frac{1/x}{1} = \lim_{x \rightarrow \infty} \frac{1}{x}.$$

From basic limits,

$$\lim_{x \rightarrow \infty} \frac{1}{x} = 0.$$

Therefore,

$$\lim_{x \rightarrow \infty} \frac{\ln x}{x} = 0.$$

$$\boxed{0}$$