

Question 8

Prove that the volume of a right circular cylinder with radius R and height h is

$$V = \pi R^2 h.$$

Question 8 - Solution

We derive the volume formula using cross-sectional areas and integration.

Place the cylinder so that its base lies in the xy -plane and its axis is along the x -axis. Then the cylinder extends from $x = 0$ to $x = h$.

At any position x with $0 \leq x \leq h$, a cross-section perpendicular to the x -axis is a circle of radius R .

The area of this circular cross-section is constant and equal to

$$A(x) = \pi R^2.$$

The volume of the cylinder can be approximated by slicing it into thin slabs of thickness dx . Each slab has volume approximately

$$dV = A(x) dx = \pi R^2 dx.$$

Adding the volumes of all such slabs gives

$$V = \int_0^h \pi R^2 dx.$$

Since πR^2 is constant, evaluate the integral:

$$V = \pi R^2 \int_0^h dx = \pi R^2 [x]_0^h = \pi R^2 h.$$

$$\boxed{V = \pi R^2 h}$$