

Question 3

Assume that f is integrable on $[a, b]$ and that $a < b$. Prove that

$$\int_a^a f(x) \, dx = 0.$$

Question 3 - Solution

We use the definition of the definite integral in terms of Riemann sums.

Consider the interval $[a, a]$. Any partition of this interval consists of a single point:

$$a = x_0 = x_1.$$

The width of the subinterval is

$$\Delta x = x_1 - x_0 = a - a = 0.$$

Choose any sample point x^* in the interval $[a, a]$.

A Riemann sum for f over $[a, a]$ is

$$\sum f(x^*) \Delta x = f(x^*) \cdot 0 = 0.$$

Since every Riemann sum over $[a, a]$ equals 0, the limit of the Riemann sums exists and equals 0.

By definition of the definite integral,

$$\int_a^a f(x) dx = 0.$$

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