

Question 8

Assume that f is differentiable on an open interval I containing a , and that

$$f'(a) = 0 \quad \text{and} \quad f''(a) > 0.$$

Prove that f has a strict local minimum at $x = a$.

Question 8 - Solution

By the definition of the second derivative and $f'(a) = 0$,

$$\lim_{h \rightarrow 0} \frac{f'(a+h) - f'(a)}{h} = f''(a).$$

Because this limit is nonzero, the quotient has the same sign as $f''(a)$ for all sufficiently small nonzero h .

Consequently, $f'(a+h) < 0$ for negative h and $f'(a+h) > 0$ for positive h . Thus f is strictly decreasing just left of a and strictly increasing just right of a , by the Mean Value Theorem.

Therefore f has a strict local minimum at a . No continuity assumption on f'' near a is needed.