

Question 5

Assume that f is differentiable on an interval (a, b) and that

$$f'(x) < 0 \quad \text{for all } x \in (a, b).$$

Prove that f is decreasing on (a, b) .

Question 5 - Solution

To prove that f is decreasing on (a, b) , we must show that for any $x_1, x_2 \in (a, b)$ with $x_1 < x_2$, the inequality

$$f(x_1) > f(x_2)$$

holds.

Let $x_1, x_2 \in (a, b)$ with $x_1 < x_2$. Because f is differentiable on (a, b) , it is continuous on $[x_1, x_2]$ and differentiable on (x_1, x_2) .

By the Mean Value Theorem, there exists a number $c \in (x_1, x_2)$ such that

$$f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

By assumption, $f'(c) < 0$, and since $x_2 - x_1 > 0$, it follows that

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} < 0.$$

Multiply both sides by $x_2 - x_1$:

$$f(x_2) - f(x_1) < 0.$$

Thus,

$$f(x_2) < f(x_1).$$

Since this holds for all $x_1 < x_2$ in (a, b) , the function f is decreasing on (a, b) .

f is decreasing on (a, b)