

**Question 9**

Prove that

$$\lim_{x \rightarrow 0} \frac{\sin(2x)}{\tan x} = 2.$$

**Question 9 - Solution**

We begin by rewriting the expression using trigonometric identities.

Recall that

$$\tan x = \frac{\sin x}{\cos x}.$$

Substitute this into the fraction:

$$\frac{\sin(2x)}{\tan x} = \sin(2x) \cdot \frac{\cos x}{\sin x}.$$

Next, use the identity

$$\sin(2x) = 2 \sin x \cos x.$$

Substitute:

$$\frac{\sin(2x)}{\tan x} = (2 \sin x \cos x) \cdot \frac{\cos x}{\sin x}.$$

Cancel  $\sin x$  (for  $x \neq 0$ ):

$$\frac{\sin(2x)}{\tan x} = 2 \cos^2 x.$$

Now take the limit as  $x \rightarrow 0$ .

Since cosine is continuous,

$$\lim_{x \rightarrow 0} \cos x = \cos 0 = 1.$$

Therefore,

$$\lim_{x \rightarrow 0} 2 \cos^2 x = 2(1)^2 = 2.$$

$\boxed{2}$