

Question 8

Prove that

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0.$$

Question 8 - Solution

We begin by rewriting the expression using a trigonometric identity.

Recall that

$$\cos x - 1 = -(1 - \cos x).$$

Thus,

$$\frac{\cos x - 1}{x} = -\frac{1 - \cos x}{x}.$$

From a previously established trigonometric limit,

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0.$$

Therefore,

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = -\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = -0 = 0.$$

$$\boxed{0}$$